Exercises for the PhD course Graph Theory

Lecture 1

1. Prove or disprove: If a (simple) graph with diameter 2 has a cut-vertex, then its complement has an isolated vertex.

2. (a) Show that if G is a connected graph where all vertex degrees are even, then for any $v \in V(G)$ $c(G - v) \leq \frac{1}{2} d_G(v)$.

   (b) Let G be a connected graph with $|V(G)| \geq 3$. Show that if G has a cut-edge, then G has a vertex v such that $c(G - v) > c(G)$, while the converse is not necessarily true.

3. Prove that every connected graph G contains a path of length at least $\min\{2\delta(G), |V(G)| - 1\}$.