

# Exercises for the PhD course Graph Theory

## Lecture 10

1. Show that  $t_{r-1}(n)/\binom{n}{2}$  converges to  $(r-2)/(r-1)$  as  $n \rightarrow \infty$ .  
(Hint:  $t_{r-1}((r-1)\lfloor \frac{n}{r-1} \rfloor) \leq t_{r-1}(n) \leq t_{r-1}((r-1)\lceil \frac{n}{r-1} \rceil)$ .)
2. (a) Show that Hadwiger's conjecture for  $r+1$  implies the conjecture for  $r$ .  
(b) Prove the following weakening of Hadwiger's conjecture: given any  $\epsilon > 0$ , every graph of chromatic number at least  $r^{1+\epsilon}$  has a  $K_r$  minor, provided that  $r$  is large enough.
3. (a) Prove that if  $G$  has minimum degree at least 3, then  $G$  contains a (topological)  $K_4$  minor.  
(b) A multigraph is called *series-parallel* if it can be constructed recursively from a  $K_2$  by the operations of subdividing and doubling edges. Show that a 2-connected multigraph is series-parallel if and only if it has no (topological)  $K_4$  minor.