Exercises for the PhD course Graph Theory

Lecture 12

1. (a) Let $\epsilon > 0$ and $p = p(n) > 0$, and let $r \geq (1+\epsilon)(2\log n)/p$ be an integer-valued function of n. Show that almost no graph in $\mathcal{G}(n,p)$ contains r independent vertices. (Here, log is the natural logarithm.)

   (b) Show that for every $0 < \epsilon < 1$ and $p = (1-\epsilon)\frac{\log n}{n}$ almost every graph in $\mathcal{G}(n,p)$ has an isolated vertex. Hint: Use the second moment method (Lemma 11.4.2).

2. Show that for constant $p \in (0,1)$ almost every graph in $\mathcal{G}(n,p)$ has diameter 2.

3. Show that for every graph H there is a function $p = p(n)$ such that $p \to 0$ as $n \to \infty$, but almost every graph in $\mathcal{G}(n,p)$ contains an induced copy of H.