

Exercises for the PhD course Graph Theory

Lecture 2

1. Let G be a (simple) connected graph not having P_4 (the path on four vertices) or C_3 (cycle on three vertices) as an induced subgraph. Prove that G is complete bipartite.
2. (a) Prove that every connected graph has a orientation in which the number of vertices with odd outdegree is at most 1. (The outdegree of a vertex v in a digraph is the number of arcs leaving v).
(b) Use part (a) to conclude that a (simple) connected graph with an even number of edges can be decomposed into paths with two edges. (Recall that a decomposition of G is a collection of edge-disjoint subgraphs of G such that each edge of G is in precisely one of these graphs.)
3. Let G be a tree with $2k$ vertices of odd degree. Prove that G decomposes into k paths.