Exercises for the PhD course Graph Theory

Lecture 3

1. A bipartite graph is called $(3,6)$-biregular is all vertices in one part X have degree 3, and all vertices in the other part Y have degree 6. Prove that every $(3,6)$-biregular graph has a spanning subgraph where every vertex of X has degree 1 and every vertex in Y has degree 2.

2. Prove that a (simple) 3-regular graph has a 1-factor if and only if it decomposes into copies of P_4.

3. (a) Find a bipartite graph and a set of preferences such that no matching of maximal size is stable and no stable matching has maximal size.

   (b) Find a non-bipartite graph with a set of preferences that has no stable matching.