Exercises for the PhD course Graph Theory

Lecture 4

1. For each $k > 1$, construct a k-regular (simple) graph with no 1-factor.
2. A graph G is called vertex-transitive if, for any two vertices $v, w \in V(G)$, there is an automorphism of G mapping v to w. Using the observations following the proof of Theorem 2.2.3, show that every vertex-transitive connected graph of even order contains a 1-factor.
3. Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$.