Exercises for the PhD course Graph Theory

Lecture 8

1. Show that $\chi(G) \in \{\omega(G), \omega(G)+1\}$ for every line graph G.

2. (a) Show that if G is a graph, then G is contained in a $\Delta(G)$-regular graph (without loops and multiple edges).

   (b) Using (a), and Petersen's 2-factor theorem, prove that $\chi'(G) \leq 3\left\lceil \frac{\Delta(G)}{2} \right\rceil$ for any graph G.

3. Show (without using Lovasz' Theorem 5.5.4) that the complement of any bipartite graph is perfect. (Hint: use König's theorem 2.1.1 on minimum vertex covers.)