

Exercises for the PhD course Graph Theory

Lecture 9

1. For every positive integer $k > 1$, find a 2-chromatic graph whose choice number is k .
2. (a) Find a general upper bound on $\chi'_l(G)$ in terms of $\chi'(G)$.
(b) Without using Theorem 5.4.2, prove that every planar graph is 6-list-colorable.
3. The total chromatic number $\chi''(G)$ of G is the least number of colors needed for a coloring $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots\}$ so that any adjacent or incident pair of elements of $V(G) \cup E(G)$ are assigned different colors. The total coloring conjecture suggests that $\chi''(G) \leq \Delta(G) + 2$ for any graph G . Bound the total chromatic number from above in terms of the list-chromatic index, and use this bound to deduce a weakening of the total coloring conjecture from the list coloring conjecture.