MAI0106 Nov-Dec 2011

Assignment 2

Consider the differential equation

$$u_t + Au_x = F \ , \quad 0 \le x < \infty \, , \ 0 \le t \, ,$$

where u and F are vector functions with two components, and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
The initial and boundary conditions are

$$\begin{aligned} u(x,0) &= f(x) \, , \\ u^{(1)}(0,t) + \alpha u^{(2)}(0,t) &= g(t) \, , \\ ||u|| &< \infty \, . \end{aligned}$$

Approximate the problem by

$$\begin{aligned} \frac{du_j}{dt} + AD_0 u_j &= F_j \ , \quad j = 1, 2, \dots \, , \\ u_j(0) &= f_j \, , \\ u_0^{(1)}(t) + \alpha u_0^{(2)}(t) &= g(t) \, , \\ \frac{du_0^{(2)}}{dt} + D_+ u_0^{(1)} &= F_0^{(2)} \, , \\ ||u||_h &< \infty \, . \end{aligned}$$

1. Derive the Lopatinsky and the Kreiss condition for the IBVP problem. Investigate when they are satified. For which, if any, α-values can we conclude strong well-posedness ? Note that no interval for the α-values is given.

2. Derive the Godunov-Ryabenkii and the Kreiss condition for the numerical approximation of the IBVP problem and investigate whether or not they are both satisfied for all α in the interval $0 \le \alpha \le 1$. For which α-values can we conclude strong stability?

3. **Challenge:** Use the normal mode analysis and derive a strongly well-posed energy-estimate for the IBVP. The estimate must contain all the data F, f, g. You can pick a suitable α if you want.

Motivate your answers clearly !