CME MAI0106 Nov-Dec 2011

Assignment 4

Consider the partial differential equation

$$u_t = u_{xx} + F(x,t)\ , \quad u(x,0) = f(x), \quad 0 \le x \le 2\pi. \tag{1}$$

1. Find the periodic solution for the case

   $$f(x) = \sin(\omega x)\,,$$
   $$F(x,t) = (\omega^2 - \alpha)\sin(\omega x)e^{-\alpha t}\,,$$

   where ω and α are constants. (Make the ansatz $u(x,t) = a\sin(\omega x)e^{-bt}$.)

2. Prove that BDF2 (see (5.12) in course reader) is both A-stable and L-stable when applied to the test equation (5.2).

3. Discretize in space by using the 2nd and 4th order difference operators in Table 4.1, and also the 4th order Padé type operator in Table 4.5. Denote these operators by Q_1, Q_2, Q_3. Write a program that solves the problem by using BDF2 on the problem

   $$u_t = Q_\nu u + F\,, \quad u(0) = f, \qquad \nu = 1, 2, 3\,.$$

4. Choose $\omega = 5$ and $\alpha = 1$ and run the program with the number of points in space determined by Table 1.2. Measure the error at $t = 1$ in the max-norm, and comment on the agreement with the table. The time intergration method is 2nd order accurate, which means that one has to choose a time step to match the space accuracy. Let me know your choices.

5. **Challange:** Discretise problem (2) in *Assignment 3* on the domain $0 \le x \le 1$ using central differences (D_0) in the interiour and the approximations below at the boundaries. Eq. (2) is the numerical outflow approximation. Eq. (3) approximate the PDE at the first point close to the inflow boundary using $u_0 = g_1(t)$ from (2)(a) in *Assignment 3*.

   Prove that the approximation is second order accurate despite the first order error at the outflow boundary. (Remember the interiour and boundary error equations.) You can work with $\theta = -1$ and view the boundary approximation at $x = 1$ as a halfplane problem.

   $$(u_N)_t = -(u_N - u_{N-1})/h + \theta u_N, \tag{2}$$
   $$(u_1)_t = -(u_2 - g_1(t))/(2h) + \theta u_1, \tag{3}$$

Motivate your answers clearly !