MAI0106 Nov-Dec 2011

Assignment 6

Consider again the scalar wave propagation problem,

$$\begin{aligned} u_t + au_x + bu_y &= 0, \quad (x,y) \in \Omega, \\ Lu &= g(x,y,t), \quad (x,y) \in \delta\Omega \\ u(x,y,0) &= f(x,y), \quad (x,y) \in \Omega. \end{aligned} \tag{1}$$

The wave propagation direction $\bar{a} = (a, b)$ is constant and both a and b are postive. The domain Ω has an outward pointing normal $\bar{n}$

1. Discretize (1) using a discontinuous Galerkin (dG) method. Show how to make it conservative and stable at the element interfaces. Show also how to implement the boundary conditions in a stable way.

2. Show how the dG method can be used to derive a finite volume method (FVM). Is the FVM cell-centered or node-centered ? Hint: solutions to FVM are constant in each volume.

3. **Challange:** Both the FDM used in the previous assignment and dG methods above are on a SBP form and impose boundary and interface conditions weakly. Can these schemes be combined in a stable and accurate way ? *Discuss* how that could be done at a common smooth interface. Use concepts like energy estimates, interpolation operators, scalar products, norms, penalty formulation, etc. The two central themes of this course, namely accuracy and stability must be mentioned. The more details, the bettter..., but no fairy tales.

4. Choose your favourite research issue from the material in this course, think through how that research would be done and be prepared to **present that in 5-10 minutes** to the other participants in the course. The date for the preentation will be given later. You are not allowed to pick the task in item 3 above.

Motivate your answers clearly !