MAI0142
Hand in Problems – 12

1. Let $\boldsymbol{x}_{ij}$ be independent and identically distributed vectors with p-variate normal distribution $N_p(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$, where $i = 1, 2$ and $j = 1, ..., N_i$. For simplicity let $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \boldsymbol{I}_p$.
   (a) Simulate the ASL for the tests Hotelling T^2, Dempster (1958), Bai and Sarandasa (1996), Srivastava (2007) and Srivastava and Du (2008), discussed in Lecture 12 for the combinations $N_1 = N_2 = \{15, 30\}$ and $p = \{5, 10, 50, 100, 200\}$ (when possible). Give a table. Conclusion?
   (b) Simulate also the power for the same tests and p and N_1, N_2. Choose an alternative, i.e., $\boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_2$, that give resonable power. Give a table. Conclusion?

2. Consider the covariance matrices for the three different estimators for the parameter matrix $\boldsymbol{B}$ in the GCM discussed in Lecture 12, i.e.,

$$\begin{aligned}
\text{cov}\left(\widehat{\boldsymbol{B}}_{MLE}\right) &= \frac{n-1}{n-1-(p-q)}(\boldsymbol{C}\boldsymbol{C}')^{-1} \otimes (\boldsymbol{A}'\boldsymbol{\Sigma}^{-1}\boldsymbol{A})^{-1}, \\
\text{cov}\left(\widehat{\boldsymbol{B}}\right) &= (\boldsymbol{C}\boldsymbol{C}')^{-1} \otimes (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{\Sigma}\boldsymbol{A}(\boldsymbol{A}'\boldsymbol{A})^{-1}. \\
\text{cov}\left(\widehat{\boldsymbol{B}}_{AL}\right) &\approx \frac{(p-q-1)(p-1)}{(n-q-1)(p-n+q-1)}(\boldsymbol{C}\boldsymbol{C}')^{-1} \otimes (\boldsymbol{A}'\boldsymbol{\Sigma}^{-1}\boldsymbol{A})^{-1}.
\end{aligned}$$

   Compare these covariance matrices for the cases
   (i) $n \gg p$,
   (ii) $n \gtrsim p$,
   (iii) $n = p$,
   (iv) $n \lesssim p$,
   (v) $n \ll p$.

   How about the size of q? Also discuss the role of the matrices $(\boldsymbol{A}'\boldsymbol{\Sigma}^{-1}\boldsymbol{A})^{-1}$ versus $(\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{\Sigma}\boldsymbol{A}(\boldsymbol{A}'\boldsymbol{A})^{-1}$.