MAI0142
Hand in Problems – 6

1. Show that the LRT for testing the hypothesis H_1 (*similarity of profiles*) for two groups is given by

$$\frac{n_1 + n_2 - p}{(n_1 + n_2 - 2)(p - 1)} \frac{n_1 n_2}{n_1 + n_2} \boldsymbol{u}' \boldsymbol{C}' (\boldsymbol{C}\boldsymbol{S}_p \boldsymbol{C}')^{-1} \boldsymbol{C}\boldsymbol{u} \geq c_\alpha,$$

   where c_α is chosen so the size of the test is α. $\Big[$Optional: show that $c_\alpha = F_\alpha(p - 1, n_1 + n_2 - p).\Big]$

2. Let $\boldsymbol{x}_{ij}$ be p-dimensional random vectors independent distributed as $\boldsymbol{x}_{ij} \sim N_p(\boldsymbol{\mu}_i, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}_i = (\mu_{i1}, \ldots, \mu_{ip})'$, $\boldsymbol{\Sigma} > 0$, $j = 1, \ldots, n_i$, $i = 1, \ldots, k$ and $N = n_1 + \cdots + n_k$. Derive the likelihood function (given on Lecture 6, slide 8) and the MLEs under the hypothesis $H_1 : \boldsymbol{\mu}_i - \boldsymbol{\mu}_k = \gamma_i \boldsymbol{1}_p$, $i = 1, \ldots, k - 1$ and the alternative $A_1 \neq H_1$ (Lecture 6, slide 10).

3. Psychomotor scores were obtained for subjects during three days after radiation treatment. The subjects were assigned to one of four treatment groups.

	Day				Day		
Subject	1	2	3	Subject	1	2	3
	Control				87.5r		
1	223	242	238	21	206	199	237
2	72	81	66	22	208	222	237
3	172	214	239	23	224	224	261
4	171	191	203	24	119	149	196
5	138	204	213	25	144	169	164
6	22	24	24	26	170	202	181
	37.5r			27	93	122	145
7	53	102	104	28	237	243	281
8	45	50	54	29	208	235	249
9	47	45	34	30	187	199	205
10	167	188	209	31	95	102	96
11	183	206	210	32	46	67	28
12	91	154	152	33	95	137	99
13	115	133	136	34	59	76	101
14	32	97	86	35	186	198	201
15	37	38	40		187.5r		
16	66	131	148	36	202	229	232
17	210	221	251	37	126	159	157
18	167	172	212	38	54	75	75
19	23	18	30	39	158	168	175
20	234	260	269	40	175	217	235
				41	147	183	181
				42	105	107	92
				43	213	263	260
				44	258	248	257
				45	257	269	270

   (a) Plot the profiles.
   (b) Test if the profiles of the groups are parallel.
   (c) If the profiles are parallel, are they at the same level?
   (d) If the profiles are parallel, are they flat?

   Conclusions?