Comments on *A Short Course on Spectral Theory* by W. Arveson

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These notes are comments on the book Arveson, W., *A Short Course on Spectral Theory*, Springer-Verlag, New York, 2002. Things pointed out in the errata of June, 2003, are not included here.

page 11. Line 2: delete extra ")".

page 19. Line 8: "... there is a ...".

page 20. Line 16: $|\lambda|^n \|x\|^n = \|\lambda x\|^n$ should be replaced by $|\lambda|^n \|x^n\| = \|(\lambda x)^n\|$

page 22. Line -5: the word *bounded* should be deleted. (It is not assumed elsewhere that general homomorphisms are bounded.)

page 23. Comment to lines 5–9: This proof also shows that $\|\pi\| = 1$ if $\omega \neq 0$.

page 23. Comment to lines 11–13: It is nowadays more common to also assume that $x \leq x$ for elements x in a partially ordered set. For the purpose here either definition can be used without any difference in the conclusions.

page 24. Lines 1–3: The first sentence of this proof can be replaced by a reference to Proposition 1.8.2.

page 24. Line 14: "... N is any proper ideal ...".

page 29. Line 2: "... $f_k(p) = \omega(f_k)$ ...".

page 29. Lines 7–9: A shortening can be made as follows (this best fits at the end of the previous paragraph): "Thus $g(X) \subset [\epsilon, \infty)$ and by Lemma 1.10.3, $\omega(g) \in [\epsilon, \infty)$, in particular $\omega(g) > 0$."

page 32. Line 25: "... in A^{-1}, it converges ...".

page 34. Line 24: A ")" should be deleted in the second integral.

page 41. Comment to line 25: An operator A is *self-adjoint* if $A = A^*$.

page 41. Comment to lines 26–27: See comment to page 75, lines 8–9.

page 41. Comment to line 29: The operators called projections here are often called orthogonal projections in other parts of the literature.

page 42. Line 3: "... operation $A \mapsto A^*$."

page 43. Line 27: "... commutative unital C^*-algebra ...". (That the C^*-algebra is unital is proved explicitly in the proof.)

page 46. (Line 8: I would have written "A C^*-algebra A is a Banach algebra that ...".)

page 49. Line 5: Replace $\mathrm{sp}_A(x)$ by $\sigma_A(x)$.

page 50. Line 13: "... T is a non-zero scalar ...". (It is also possible to write "positive" instead of non-zero.)

Errata to page 51. (I do not understand this comment in the errata.)

page 54. Comment to line -3: The Riesz–Markov theorem is called the Riesz representation theory in many books. (A reference to this theorem is also made on p. 61, line -5.)

page 57. Line 18. "$*$-representation" should be "representation".

Errata to page 59. Line 2: "second next item".

page 60. (Line -5: To be clearer this can be rephrased as "... interpreted as an absolutely convergent ...".

page 63. Line 6 and 9: "the bounded convergence theorem" may be replaced by "Lebesgue's dominated convergence theorem" on both lines.

page 65. Line 23: "$E \in \mathcal{B} \to P(E) \in \mathcal{B}(H)$" may be replaced by "$E \in \mathcal{B} \mapsto P(E) \in \mathcal{B}(H)$"

page 65. Line -7: "$E \subseteq F \implies P(E) \leq P(F)$" (or make an alternative change in the proof).

page 70. Line -2: "positive" may be replaced by "non-negative".

page 70. Line -1: Replace $(0, +\infty]$ by $[0, +\infty]$.

page 72. Comment to line 9: It should be observed that $A^*A \geq 0$ always, which is trivial to prove.

page 72. Line -9: "... (see Exercise (4)".

Errata to page 72. "Exeercise" should be replaced by "Exercise".

page 75. Comment to lines 8–9: Exercise (5) may be given already after Chapter 2.1, or after Chaper 2.4, when its solution together with the spectral theorem can justify the sentence on page 41, lines 26–27.

page 75. Comment to lines 11–12: The first discussion of adjoining units is in Proposition 2.5.4.

page 75. Comment to lines 30–31: This sentence could be rephrased "We will show that (a) the operator norm makes A^e into a C^*-algebra and (b) ...".

page 76. Line -4: The only x should be replaced by z.

page 84. Comment to line 11: The Čech compactification is the same as the Stone–Čech compactification on p. 81.

page 85. Comment to line 14: The right-hand side equals $\limsup_{n\to\infty} |x_n|$.

page 85. Comment to line 15: The essential spectrum is defined on the bottom of p. 93.

page 85. Line 17: "... $\sigma_e(A)\}$."

page 90. Line 8: Replace e_{n+1} by e_{n+k}.

page 92. Line 6: Replace e_n by u_n.

page 98. Comment to line 1: The line could have started PROOF OF THEOREM 3.4.1.

page 99. Comment to lines 16–17: One should require n or n_0 to be so large that $A_n \in \mathcal{F}(E)$.

page 133. Essential spectrum is started to be defined on p. 93.