Kortfattade lösningsförslag till tentamen 764G07 del 2, 2019-01-16

1a $\displaystyle\int_1^2 \frac{2}{x^2+2x}\,dx = \int_1^2 \frac{2}{x(x+2)}\,dx = \int_1^2 \left(\frac{1}{x} - \frac{1}{x+2}\right)dx =$

$= \left(\ln\left|\frac{x}{x+2}\right|\right)_1^2 = \ln\frac{2}{4} - \ln\frac{1}{3} = \ln\frac{3}{2}$

1b $\displaystyle\int e^{\sin x}\cos x\,dx \;\Big/ \begin{matrix} t = \sin x \\ dt = \cos x\,dx \end{matrix} \Big/ = \int e^t\,dt =$

$= e^{\sin x} + C$

1c $\displaystyle\int x^5 \cos x^3\,dx \;\Big/ \begin{matrix} t = x^3 \\ dt = 3x^2\,dx \end{matrix} \Big/ = \frac{1}{3}\int t\cos t\,dt$

$\Big/ \begin{matrix} \text{PI} & \\ g = t & g' = 1 \\ f = \cos t & F = \sin t \end{matrix} \Big/ = \frac{1}{3}\left(t\sin t - \int \sin t\,dt\right) =$

$= \frac{1}{3}t\sin t + \frac{1}{3}\cos t + C = \frac{1}{3}x^3\sin x^3 + \frac{1}{3}\cos x^3 + C$

<u>Svar:</u> a) $\ln\frac{3}{2}$ b) $e^{\sin x} + C$ c) $\frac{1}{3}x^3\sin x^3 + \frac{1}{3}\cos x^3 + C$.

2 $y' = y\dfrac{x}{3+x^2} \Leftrightarrow \dfrac{dy}{y} = \dfrac{x}{3+x^2}dx$, $y \neq 0$; $y = 0$ – en lösn.

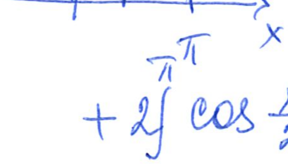
$\ln|y| = \frac{1}{2}\ln(3+x^2) + \ln C$, $C > 0 \Leftrightarrow y = C\sqrt{3+x^2}$, $C \neq 0$

samtliga lösningar: $y = C\sqrt{3+x^2}$, $C \in \mathbb{R}$

$y(1) = 4 \Rightarrow 4 = 2C \Rightarrow C = 2$

<u>Svar:</u> $y = 2\sqrt{3+x^2}$.

3



$dV = 2\pi x \cdot f(x)\,dx \Rightarrow V = 2\pi\displaystyle\int_0^{\pi} x\sin\frac{x}{2}\,dx$

$\Big/ \begin{matrix} \text{PI} & \\ g = x & g' = 1 \\ f = \sin\frac{x}{2} & F = -2\cos\frac{x}{2} \end{matrix} \Big/ = 2\pi\left(\left(-2x\cos\frac{x}{2}\right)_0^{\pi} + \right.$

$\left. + 2\displaystyle\int_0^{\pi}\cos\frac{x}{2}\,dx\right) = 2\pi\left(4\sin\frac{x}{2}\right)_0^{\pi} = 8\pi$ (v.e.)

Svar: $V = 8\pi$ v.e.

4a $\displaystyle\lim_{x\to 0}\frac{\sin(2x) - 2x}{x(1-\cos x)} = \lim_{x\to 0}\frac{2x - \frac{(2x)^3}{3!} + O(x^5) - 2x}{x\left(1 - \left(1 - \frac{x^2}{2} + O(x^4)\right)\right)} =$

$$= \lim_{x\to 0} \frac{-\frac{4}{3}x^3+O(x^5)}{\frac{x^3}{2}+O(x^5)} = \lim_{x\to 0}\frac{x^3\left(-\frac{4}{3}+O(x^2)\right)}{x^3\left(\frac{1}{2}+O(x^2)\right)} = -\frac{8}{3}$$

4b
$$\lim_{x\to 1}\frac{(x^{1/3}-1)(x-1)}{\arctan(2x-2)^2} \;\Big/\; \begin{matrix} t=x-1 \\ t\to 0\end{matrix} \Big/ = \lim_{t\to 0}\frac{((1+t)^{1/3}-1)\,t}{\arctan(2t)^2} =$$

$$= \lim_{t\to 0}\frac{t\left(\cancel{1}+\frac{1}{3}t+O(t^2)-\cancel{1}\right)}{4t^2+O(t^6)} = \lim_{t\to 0}\frac{\cancel{t^2}\left(\frac{1}{3}+O(t)\right)}{\cancel{t^2}\left(4+O(t^4)\right)} = \frac{1}{12}$$

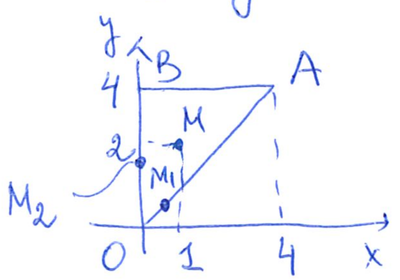
4c
$$\int_0^\infty \frac{dx}{4+9x^2} = \lim_{b\to\infty}\ \frac{2}{3}\cdot\frac{1}{4}\int_0^b \frac{\frac{3}{2}\,dx}{1+\left(\frac{3}{2}x\right)^2} = \frac{1}{6}\lim_{b\to\infty}\left(\arctan\tfrac{3}{2}x\right)\Big|_0^b =$$

$$= \frac{1}{6}\lim_{b\to\infty}\left(\arctan\tfrac{3}{2}b - 0\right) = \frac{\pi}{12}$$

Svar: a) $-\frac{8}{3}$ b) $\frac{1}{12}$ c) $\frac{\pi}{12}$

5
$$f(x,y) = x^2+xy-4x-y^2+3y-2$$

$$\begin{cases} f'_x = 2x+y-4=0 \\ f'_y = x-2y+3=0\end{cases} \Rightarrow \begin{matrix} x=1 \\ y=2\end{matrix} \Rightarrow M=(1,2)$$



$D:\ 0\le x\le y\le 4$

$f(1,2) = -1$

$OA:\ y=x \Rightarrow f(x,x) = x^2-x-2, \quad 0\le x\le 4$

$f'(x,x) = 2x-1=0 \Rightarrow x=\frac{1}{2} \qquad M_1=\left(\frac{1}{2},\frac{1}{2}\right)$

$f\left(\frac{1}{2},\frac{1}{2}\right) = -\frac{9}{4}$

$BA:\ y=4 \Rightarrow f(x,4) = x^2-6, \quad 0\le x\le 4)$

$f'(x,4) = 2x=0 \Leftrightarrow x=0$

$f(0,0)=-2$

$f(4,4)=10$

$OB:\ x=0 \Rightarrow f(0,y) = -y^2+3y-2, \quad 0\le y\le 4$

$f'(0,y) = -2y+3=0 \Leftrightarrow y=\frac{3}{2} \qquad M_2=\left(0,\frac{3}{2}\right)$

$f(M_2) = f\left(0,\frac{3}{2}\right) = \frac{1}{4} \qquad f(0,4)=-6$

Svar: $f_{max} = f(4,4) = 10, \ f_{min} = f(0,4) = -6$

(6)
$$y + \int_0^x \frac{y(t)}{1+t^2}\,dt = 2\arctan x$$

$$\Leftrightarrow \begin{cases} y' + \dfrac{y}{1+x^2} = \dfrac{2}{1+x^2} \Rightarrow IF = e^{\arctan x} \\ y(0) = 0 \end{cases}$$

$$y' \cdot e^{\arctan x} + e^{\arctan x} \cdot \frac{1}{1+x^2} \cdot y = \frac{2}{1+x^2} e^{\arctan x}$$

$$\left(y e^{\arctan x}\right)' = \frac{2}{1+x^2} e^{\arctan x}$$

$$y e^{\arctan x} = 2\int e^{\arctan x} \frac{dx}{1+x^2} \Bigg/ \begin{matrix} t = \arctan x \\ dt = \dfrac{dx}{1+x^2} \end{matrix} \Bigg/ =$$

$$= 2\int e^t dt = 2e^{\arctan x} + C \Rightarrow$$

$$y = 2 + C e^{-\arctan x}, \quad y(0) = 0 \Rightarrow C = -2$$

Svar: $y = 2 - 2e^{-\arctan x}$

(7)
$$r = \frac{3}{7}\varphi^2, \quad 0 \le \varphi \le a \qquad \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$$ds = \sqrt{r^2 + r'^2}\,d\varphi = \frac{3}{7}\sqrt{\varphi^4 + 4\varphi^2}\,d\varphi$$

$$s = \int_0^a ds = \frac{3}{7}\int_0^a \sqrt{\varphi^4 + 4\varphi^2}\,d\varphi. \quad s = 8 \Rightarrow$$

$$8 = \frac{3}{7}\int_0^a \sqrt{\varphi^2+4}\cdot\varphi\,d\varphi \Bigg/ \begin{matrix} t = 4+\varphi^2 \\ dt = 2\varphi d\varphi \\ \varphi = 0 \Rightarrow \alpha = 4 \\ \varphi = a \Rightarrow \beta = 4+a^2 \end{matrix} \Bigg/ =$$

$$= \frac{3}{7}\cdot\frac{1}{2}\cdot\frac{2}{3}\left((4+\varphi^2)^{3/2}\right)\Big|_0^a = \frac{1}{7}\left((4+a^2)^{3/2} - 8\right) \Leftrightarrow$$

$$7\cdot 8 + 8 = (4+a^2)^{3/2} \Leftrightarrow 2^6 = (4+a^2)^{3/2} \Leftrightarrow 2^{6\cdot\frac{2}{3}} = 4+a^2$$

$$\Leftrightarrow 16 = 4 + a^2 \Leftrightarrow a^2 = 12 \Rightarrow a = 2\sqrt{3} \text{ ty } a \ge 0$$

Svar: $a = 2\sqrt{3}$.