Lösningsskiss till tentamen i Matematisk analys, del 2, 764G07, TEN-2, 2020-03-16

1a $\displaystyle\int_0^{\pi} x\sin 2x\,dx = \left/ \begin{matrix} g = x & \Rightarrow g' = 1 \\ f = \sin 2x & \Rightarrow F = -\frac{\cos 2x}{2} \end{matrix} \right/ = \left(-\frac{x}{2}\cos 2x\right)_0^{\pi} +$

$\displaystyle + \frac{1}{2}\int_0^{\pi} \cos 2x\,dx = -\frac{\pi}{2} + \left(\frac{1}{4}\sin 2x\right)_0^{\pi} = -\frac{\pi}{2}.$

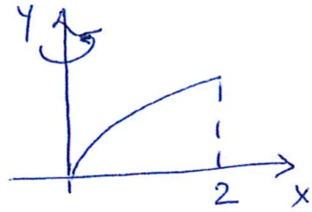
1b $\displaystyle\int_3^4 \frac{4}{x^2-4}\,dx = \int_3^4 \left(\frac{1}{x-2} - \frac{1}{x+2}\right) dx = \left(\ln|x-2| - \ln|x+2|\right)_3^4 =$

$\displaystyle = \left(\ln\left|\frac{x-2}{x+2}\right|\right)_3^4 = \ln\frac{2}{6} - \ln\frac{1}{5} = \ln\frac{5}{3}.$

1c $\displaystyle\int_0^1 \frac{x^3}{\sqrt{1+x^4}}\,dx \left/ \begin{matrix} t = 1+x^4 \\ dt = 4x^3dx \end{matrix} \right/ = \frac{1}{4}\int_1^2 \frac{1}{\sqrt{t}}\,dt = \frac{1}{4}\left(2\sqrt{t}\right)_1^2 = \frac{\sqrt{2}-1}{2}.$

$x = 0 \Rightarrow \alpha = 1$
$x = 1 \Rightarrow \beta = 2$

Svar: a) $-\frac{\pi}{2}$ b) $\ln\frac{5}{3}$ c) $\frac{\sqrt{2}-1}{2}$.

2 

$dV = 2\pi x\, f(x)\,dx \qquad V = 2\pi\displaystyle\int_0^2 x\ln(1+x^2)\,dx$

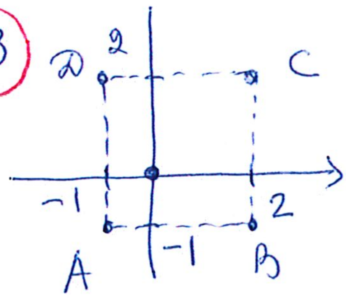
$\left/ \begin{matrix} t = 1+x^2 \\ dt = 2x\,dx \end{matrix} \right/ = 2\pi\displaystyle\int_1^5 \ln t\,\frac{dt}{2} = \pi\int_1^5 \ln t\,dt$

$x = 0 \Rightarrow \alpha = 1$
$x = 2 \Rightarrow \beta = 5$

$\left/ \begin{matrix} g = \ln t & g' = \frac{1}{t} \\ f = 1 & F = t \end{matrix} \right/ = \pi\left(\left(t\ln t\right)_1^5 - \displaystyle\int_1^5 \frac{1}{t}\cdot t\,dt\right) = \pi\left(5\ln 5 - (t)_1^5\right) =$

$= \pi(5\ln 5 - 4)$ (v.e.)

Svar: $V = \pi(5\ln 5 - 4)$

3 

$f(x,y) = 3x^2 - 2xy + 3y^2 - 7, \quad -1 \le x \le 2, \; -1 \le y \le 2.$

$\begin{cases} f'_x = 6x - 2y = 0 \\ f'_y = -2x + 6y = 0 \end{cases} \Leftrightarrow \begin{cases} 3x - y = 0 \\ -x + 3y = 0 \end{cases} \Leftrightarrow x = y = 0$

$M_1 = (0,0) \Rightarrow \boxed{f(0,0) = -7}$

Randpunkter:

AB: $y=-1 \Rightarrow f(x,-1)=3x^2+2x-4, \ -1\le x\le 2$

$f'(x,-1)=6x+2=0 \Leftrightarrow x=-\frac{1}{3}$ ligger i $[-1,2]$

BC: $x=2 \Rightarrow f(2,y)=3y^2-4y+5, \ -1\le y\le 2$

$f'(2,y)=6y-4=0 \Leftrightarrow y=\frac{2}{3}$ ligger i $[-1,2]$

DC: $y=2 \Rightarrow f(x,2)=3x^2-4x+5, \ -1\le x\le 2$

$f'(x,2)=6x-4=0 \Leftrightarrow x=\frac{2}{3}$ ligger i $[-1,2]$

AD: $x=-1 \Rightarrow f(-1,y)=3y^2+2y-4, \ -1\le y\le 2$

$f'(-1,y)=6y+2=0 \Leftrightarrow y=-\frac{1}{3}$ ligger i $[-1,2]$

$\boxed{f(-\frac{1}{3},-1)=-\frac{13}{3}}$

$\boxed{f(-1,-1)=-3}$

$\boxed{f(2,-1)=12}$

$\boxed{f(2,\frac{2}{3})=\frac{11}{3}}$

$\boxed{f(2,2)=9}$

$\boxed{f(\frac{2}{3},2)=\frac{11}{3}}$

$\boxed{f(-1,2)=12}$

$\boxed{f(-1,-\frac{1}{3})=-\frac{13}{3}}$

Svar: $f_{max}=f(2,-1)=f(-1,2)=12$

$f_{min}=f(0,0)=-7$

4a $f(x)=e^{x^2}\sin x=(1+x^2+O(x^4))(x-\frac{x^3}{3!}+O(x^5))=$

$= x+\frac{5x^3}{6}+O(x^5)$.

4b $f(x)=\begin{cases} \frac{e^{3x}-1}{\sin 5x}, & x\neq 0 \\ a, & x=0 \end{cases}$ För att f blir kontinuerlig för $x=0$

behöver vi välja $a=\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{e^{3x}-1}{\sin 5x}$.

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin 5x}=\lim_{x\to 0}\frac{1+3x+O(x^2)-1}{5x+O(x^3)}=\lim_{x\to 0}\frac{3x+O(x^2)}{5x+O(x^3)}=$$

$$=\lim_{x\to 0}\frac{x(3+O(x))}{x(5+O(x^2))}=\lim_{x\to 0}\frac{3+O(x)}{5+O(x^2)}=\frac{3}{5}$$

Svar a) $f(x)=x+\frac{5}{6}x^3+O(x^5)$ b) $\frac{3}{5}$.

5 $\frac{y'}{\cos x}-y(x)=\sin x \Leftrightarrow y'-\cos x\, y=\sin x\cdot\cos x$

$IF=e^{-\sin x} \Rightarrow (y e^{-\sin x})'=\sin x\cos x\, e^{-\sin x}$

$$y\, e^{-\sin x} = \int \sin x \cos x\, e^{-\sin x}\, dx \Big/ \begin{matrix} t = \sin x \\ dt = \cos x\, dx \end{matrix} \Big/ =$$

$$= \int t e^{-t} dt \Big/ \overset{PI}{\begin{matrix} g = t \Rightarrow g' = 1 \\ f = e^{-t} \Rightarrow F = -e^{-t} \end{matrix}} \Big/ = -te^{-t} + \int e^{-t} dt =$$

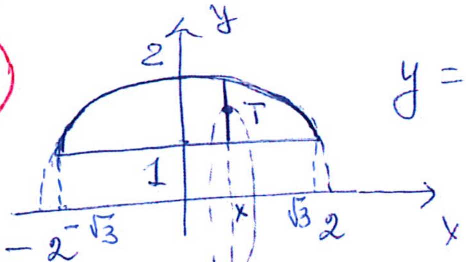
$$= -te^{-t} - e^{-t} + C = -\sin x\, e^{-\sin x} - e^{-\sin x} + C \Rightarrow$$

$$y = -\sin x - 1 + C e^{\sin x}$$

$$y(\pi) = 0 \Rightarrow 0 = -1 + C \Rightarrow C = 1 \Rightarrow \boxed{y = -\sin x - 1 + e^{\sin x}}$$

Svar

6



$$y = \sqrt{4-x^2} \qquad T = \left(x,\ \frac{1+\sqrt{4-x^2}}{2}\right)$$

$$dV = \underbrace{\left(2\pi \cdot \frac{1+\sqrt{4-x^2}}{2}\right)}_{\text{TPväg}} \cdot \underbrace{\left(\sqrt{4-x^2} - 1\right) dx}_{\text{arean}} =$$

$$= 2\pi \frac{(4-x^2)-1}{2} dx = \pi (3 - x^2)\, dx \Rightarrow V = \pi \int_{-\sqrt{3}}^{\sqrt{3}} (3-x^2)\, dx =$$

$$= \pi \left(3x - \frac{x^3}{3}\right)_{-\sqrt{3}}^{\sqrt{3}} = 4\sqrt{3}\pi \ (\text{v.e.})$$

Svar: $V = 4\pi\sqrt{3}$ v.e.

7 $\frac{2}{3} \int_1^{x^3} \frac{y(\sqrt[3]{t})}{t} dt + (x^2+1) y = 4$ Efter derivering

får vi begynnelsevärdesproblem:

$$\begin{cases} \frac{2}{3} \frac{y(x)}{x^3} \cdot 3x^2 + 2xy + (x^2+1) y' = 0 \\ 2y(1) = 4 \end{cases} \Leftrightarrow \begin{cases} y' + \frac{2}{x} y = 0 \\ y(1) = 2 \end{cases}$$

$$\frac{dy}{dx} = -\frac{2}{x} y \Rightarrow \frac{dy}{y} = -\frac{2}{x} dx,\ y \neq 0 \Rightarrow \ln|y| = -2\ln|x| + C$$

$$\Rightarrow y = C x^{-2}; \quad y(1) = 2 \Rightarrow C = 2 \Rightarrow y = \frac{2}{x^2} \text{ för } x > 0$$

Svar $y = \frac{2}{x^2}$, $x > 0$.