Lösningsskiss till tentamen i Matematisk analys del 2, 764G07, 2020-08-22

1a $\displaystyle\int_4^9 \frac{\sqrt{x}}{x-\sqrt{x}}\,dx = \left/ \begin{matrix} t=\sqrt{x} \\ x=t^2 \\ dx = 2t\,dt \\ \alpha=2,\ \beta=3 \end{matrix} \right/ = \int_2^3 \frac{t}{t^2-t}\,2t\,dt = 2\int_2^3 \frac{t}{t-1}\,dt = 2\int_2^3 \left(1+\frac{1}{t-1}\right)dt =$

$\displaystyle = 2\left(t+\ln|t-1|\right)_2^3 = 2\left(3+\ln 2-2-\ln 1\right) = 2(\ln 2+1).$

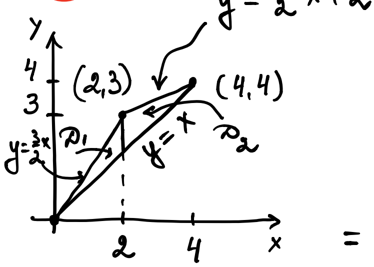
1b $\displaystyle\int x^2 \sin(x^3+1)\,dx = \left/ \begin{matrix} t=x^3+1 \\ dt=3x^2dx \end{matrix} \right/ = \frac{1}{3}\int \sin t\,dt = \frac{1}{3}(-\cos t)+C =$

$\displaystyle = -\frac{1}{3}\cos(x^3+1)+C.$

1c $\displaystyle\int \frac{6x}{x^2+x-2}\,dx = \int \frac{6x}{(x-1)(x+2)}\,dx = \int\left(\frac{2}{x-1}+\frac{4}{x+2}\right)dx = 2\ln|x-1|+4\ln|x+2|+C$

Svar: a) $2\ln 2+2$ b) $-\frac{1}{3}\cos(x^3+1)+C$ c) $2\ln|x-1|+4\ln|x+2|+C$.

2



$\displaystyle I = \iint_D (x+y)\,dx\,dy = \iint_{D_1}(x+y)\,dx\,dy + \iint_{D_2}(x+y)\,dx\,dy =$

$\displaystyle = \int_0^2\left(\int_x^{\frac{3}{2}x}(x+y)\,dy\right)dx + \int_2^4\left(\int_x^{\frac{x}{2}+2}(x+y)\,dy\right)dx =$

$\displaystyle = \int_0^2 \left(xy+\frac{1}{2}y^2\right)_{y=x}^{y=\frac{3x}{2}}dx + \int_2^4\left(xy+\frac{1}{2}y^2\right)_{y=x}^{y=\frac{x}{2}+2}dx =$

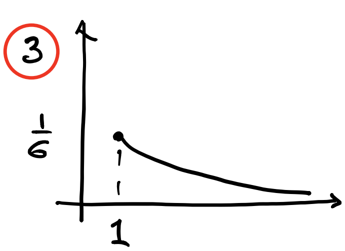
$\displaystyle = \int_0^2\left(x\cdot\frac{x}{2}+\frac{1}{2}\left(\frac{9}{4}x^2-x^2\right)\right)dx + \int_2^4\left(x\left(2-\frac{x}{2}\right)+\frac{1}{2}\left(\left(\frac{x}{2}+2\right)^2-x^2\right)\right)dx =$

$\displaystyle = \int_0^2 \frac{9}{8}x^2\,dx + \int_2^4\left(3x-\frac{7}{8}x^2+2\right)dx = \left(\frac{3}{8}x^3\right)_0^2 + \left(\frac{3}{2}x^2-\frac{7}{24}x^3+2x\right)_2^4 =$

$\displaystyle = 3+18-\frac{7}{24}\cdot 56+4 = 25-\frac{49}{3} = \frac{26}{3}.$

Svar: $I = \frac{26}{3}$.

3



$\displaystyle A = \int_1^\infty \frac{dx}{x^2+3x+2} = \lim_{b\to\infty}\int_1^b \frac{dx}{(x+1)(x+2)} =$

$$= \lim_{b\to\infty} \int_1^b \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx = \lim_{b\to\infty} \Big(\ln|x+1| - \ln|x+2|\Big)_1^b =$$

$$= \lim_{b\to\infty} \left(\ln\left|\frac{b+1}{b+2}\right| - \ln\frac{2}{3}\right) \Big/ \lim_{b\to\infty} \ln\frac{b+1}{b+2} = \lim_{b\to\infty} \underset{\text{kont}}{\ln} \left(\frac{1+\frac{1}{b}}{1+\frac{2}{b}}\right) =$$

$\to 1$

$$= \ln 1 = 0 \Big/ = -\ln\frac{2}{3} = \ln\frac{3}{2}.$$

Svar: $A = \ln\frac{3}{2}$.

4 $\quad y' + \frac{1}{x+1} y = e^{x^2+2x}, \quad x > -1$

$IF = e^{\ln(x+1)} = x+1 \Rightarrow (x+1)y' + y = (x+1)e^{x^2+2x} \Rightarrow$

$(x+1)y = \int (x+1) e^{x^2+2x}\, dx = \Big/ \begin{array}{l} t = x^2+2x \\ dt = (2x+2)dx \end{array} \Big/ = \frac{1}{2}\int e^t dt =$

$= \frac{1}{2} e^t + C = \frac{1}{2} e^{x^2+2x} + C \Rightarrow y = \frac{1}{2x+2} e^{x^2+2x} + \frac{C}{x+1}$

$y'(x) = \frac{-1}{2}(x+1)^{-2} e^{x^2+2x} + e^{x^2+2x} - \frac{C}{(x+1)^2}$

/tangenten är parallell med $y = -\frac{1}{2}x + 1$/ $\Rightarrow$

$y'(0) = -\frac{1}{2} \Rightarrow -\frac{1}{2} = -\frac{1}{2} + 1 - C \Rightarrow C = 1$

Svar: $y = \frac{e^{x^2+2x}}{2(x+1)} + \frac{1}{x+1}$.

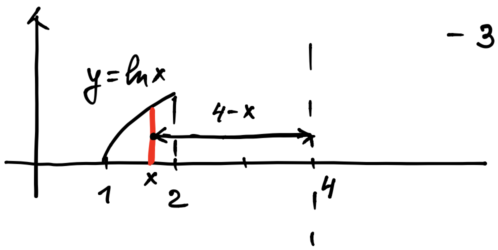
5 $\displaystyle \lim_{x\to 0} \frac{\sin ax^2 + e^{bx} - 1 - x}{\ln(1+x^2)} = \lim_{x\to 0} \frac{ax^2 + O(x^6) + 1 + bx + \frac{(bx)^2}{2} + O(x^3) - 1 - x}{x^2 + O(x^4)} =$

$$= \lim_{x\to 0} \frac{(b-1)x + x^2(a + \frac{1}{2}b^2) + O(x^3)}{x^2 + O(x^3)} = -\frac{5}{2} \Leftrightarrow \begin{cases} b - 1 = 0 \\ a + \frac{1}{2}b^2 = -\frac{5}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} b = 1 \\ a = -\frac{5}{2} - \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} b = 1 \\ a = -3 \end{cases}$$

Svar: $a = -3$, $b = 1$.

6



$$dV = \underbrace{2\pi(4-x)}_{\text{TP väg}}\, dA = 2\pi(4-x)\ln x\, dx$$

$$\Rightarrow V = \int_1^2 2\pi(4-x)\ln x\, dx \quad / \text{PI} /$$

$$= 2\pi\left[\left(-\frac{(4-x)^2}{2}\ln x\right)_1^2 + \frac{1}{2}\int_1^2 \frac{(4-x)^2}{x}\,dx\right] = 2\pi\left(-2\ln 2 + \frac{1}{2}\int_1^2\left(\frac{16}{x} - 8 + x\right)dx\right) =$$

$$= 2\pi\left(-2\ln 2 + \frac{1}{2}\left(16\ln x - 8x + \frac{x^2}{2}\right)_1^2\right) = 2\pi\left(-2\ln 2 + 8\ln 2 - 4 + \frac{3}{4}\right) =$$

$$= 2\pi\left(6\ln 2 - \frac{13}{4}\right) \text{ (v.e.)}$$

Svar: $V = 2\pi\left(6\ln 2 - \frac{13}{4}\right)$ v.e.

7

$$I = \int_{-2}^{3} g''(y)\,dy = (g')_{-2}^{3} = g'(3) - g'(-2) \quad / \; g'(b) = \frac{1}{f'(a)} \text{ där } b = f(a)$$

Antag att
$b_1 = f(a_1) = 3$
$b_2 = f(a_2) = -2$ $\Big/ \Rightarrow \quad I = \dfrac{1}{f'(a_1)} - \dfrac{1}{f'(a_2)}$

$$f(x) = e^{4x} + 2e^{2x} - 5 = (e^{2x}+1)^2 - 6$$

$$3 = f(a_1) \Leftrightarrow 3 = (e^{2a_1}+1)^2 - 6 \Leftrightarrow 9 = (e^{2a_1}+1)^2 \Leftrightarrow e^{2a_1} + 1 = \pm 3$$

-3 passar inte ty $e^{2a_1} > 0 \Rightarrow e^{2a_1} = 2$

$-2 = f(a_2) \Leftrightarrow 4 = (e^{2a_2}+1)^2 \Leftrightarrow e^{2a_2} + 1 = \pm 2$ / -2 passar inte ty $e^{2a_2} > 0$ / $\Rightarrow$

$e^{2a_2} = 1$

$$f'(x) = 4e^{4x} + 4e^{2x} \Rightarrow f'(a_1) = 4(e^{2a_1})^2 + 4(e^{2a_1}) = 4\cdot 4 + 4\cdot 2 = 24$$

$$f'(a_2) = 4(e^{2a_2})^2 + 4e^{2a_2} = 4 + 4 = 8$$

Alltså $\quad I = \dfrac{1}{24} - \dfrac{1}{8} = -\dfrac{2}{24} = -\dfrac{1}{12}$.

Svar: $I = -\dfrac{1}{12}$.