1a $\int x\sin(x^2+1)\,dx \;\Big/\begin{matrix} t=x^2+1 \\ dt=2x\,dx\end{matrix}\Big/ = \frac{1}{2}\int \sin t\,dt = -\frac{1}{2}\cos t + C =$

$= -\frac{1}{2}\cos(x^2+1)+C.$

1б $\int x e^{3x}\,dx = \Big/\begin{matrix} PI \\ g=x \Rightarrow g'=1 \\ f=e^{3x} \Rightarrow F=\frac{1}{3}e^{3x}\end{matrix}\Big/ = \frac{x}{3}e^{3x} - \frac{1}{3}\int e^{3x}\,dx =$

$= \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C.$

1c $\int \frac{\sqrt{x+1}}{x}\,dx = \Big/\begin{matrix} \sqrt{x+1}=t \\ dx=2t\,dt\end{matrix}\Big/ = \int \frac{t}{t^2-1}\cdot 2t\,dt = 2\int \frac{t^2}{t^2-1}\,dt = 2\int\left(1+\frac{1}{t^2-1}\right)dt$

$= 2t + \int\left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt = 2t + \ln|t-1| - \ln|t+1| + C =$

$= 2\sqrt{x+1} + \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C$

Svar: a) $-\frac{1}{2}\cos(x^2+1)+C$ б) $\left(\frac{x}{3}-\frac{1}{9}\right)e^{3x}+C$ c) $2\sqrt{x+1}+\ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C$

2 $y' + \frac{1}{x}y = x\sin x,\quad x>0.$

$IF = e^{\ln x} = x \Rightarrow xy' + y = x^2\sin x \Leftrightarrow (xy)' = x^2\sin x$

$\Leftrightarrow xy = \int x^2\sin x\,dx \;\Big/\begin{matrix} PI \\ g=x^2 \Rightarrow g'=2x \\ f=\sin x \Rightarrow F=-\cos x\end{matrix}\Big/ = -x^2\cos x + 2\int x\cos x\,dx =$

$= \Big/\begin{matrix} PI \\ g=x \Rightarrow g'=1 \\ f=\cos x \Rightarrow F=\sin x\end{matrix}\Big/ = -x^2\cos x + 2\left(x\sin x - \int\sin x\,dx\right) =$

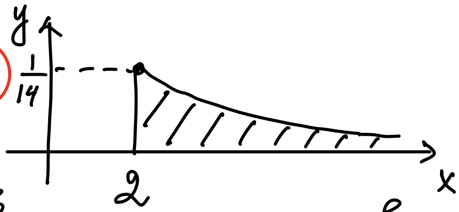
$= -x^2\cos x + 2x\sin x + 2\cos x + C \Rightarrow y = -x\cos x + 2\sin x + \frac{2}{x}\cos x + \frac{C}{x}$

− samtliga lösningar.

$y(\pi)=\pi \Rightarrow \pi = -\pi\cos\pi + 2\sin\pi + \frac{2}{\pi}\cos\pi + \frac{C}{\pi} \Leftrightarrow \pi = \pi - \frac{2}{\pi} + \frac{C}{\pi}$

$\Leftrightarrow C = 2 \Rightarrow$ Svar: $y = \left(\frac{2}{x}-x\right)\cos x + 2\sin x + \frac{2}{x}$

3



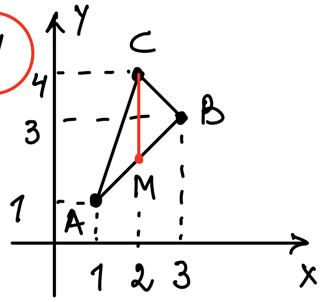
$A = \int_2^{\infty}\frac{dx}{x^2+2x+10} = \lim_{b\to\infty}\int_2^{b}\frac{dx}{(x+1)^2+9}$

$\int_2^{b}\frac{dx}{(x+1)^2+9} = \frac{1}{9}\int_2^{b}\frac{dx}{\left(\frac{x+1}{3}\right)^2+1} = \Big/\begin{matrix} t=\frac{x+1}{3} \\ dt=\frac{1}{3}dx\end{matrix}\Big/ = \frac{3}{9}\int_1^{\frac{b+1}{3}}\frac{dt}{t^2+1} = \left[\frac{1}{3}\arctan t\right]_1^{\frac{b+1}{3}} =$

$= \frac{1}{3}\left(\arctan\frac{b+1}{3} - \arctan 1\right) \to \frac{1}{3}\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \frac{\pi}{12}$

Svar: $A = \frac{\pi}{12}$ a.e.

4



$$A=\iint\limits_{\triangle ABC} dx\,dy = \iint\limits_{\triangle AMC} dx\,dy + \iint\limits_{\triangle MBC} dx\,dy$$

AB: $y=x$; AC: $y=3x-2$; BC: $y=-x+6$

$$\iint\limits_{\triangle AMC} dx\,dy = \int_1^2\left(\int_x^{3x-2} dy\right)dx = \int_1^2 (2x-2)\,dx = \left[x^2-2x\right]_1^2 =$$

$$= 0-(1-2) = 1 \text{ (a.e.)}$$

$$\iint\limits_{\triangle MBC} dx\,dy = \int_2^3\left(\int_x^{-x+6} dy\right)dx = \int_2^3 (6-2x)\,dx = \left[6x-x^2\right]_2^3 =$$

$$= 18-9-(12-4) = 1 \text{ (a.e.)} \quad \Rightarrow \quad A = (1+1)\text{ a.e.} = 2\text{ a.e.}$$

Svar: $A = 2$ a.e.

5a

$$\lim_{x\to 0}\frac{\sin 2x - \ln(1+4x)+2x}{e^{4x^2}-1} \quad \Bigg/ \begin{array}{l} e^t = 1+t+O(t^2) \\ \sin t = t + O(t^3) \\ \ln(1+t) = t - \frac{t^2}{2} + O(t^3) \end{array} \Bigg/ =$$

$$= \lim_{x\to 0}\frac{2x+O(x^3)-\left(4x-\frac{16x^2}{2}+O(x^3)\right)+2x}{1+4x^2+O(x^4)-1} = \lim_{x\to 0}\frac{8x^2+O(x^3)}{4x^2+O(x^4)} =$$

$$= \lim_{x\to 0}\frac{\cancel{x^2}(8+O(x))}{\cancel{x^2}(4+O(x^2))} = 2.$$

5b

$$\lim_{x\to\infty} x^2\left(2\ln\frac{1+x}{x} + 1 - \sqrt{\frac{x+4}{x}}\right) = \Bigg/ \begin{array}{l} t=\frac{1}{x}\to 0 \\ \text{då } x\to\infty \end{array} \Bigg/ =$$

$$= \lim_{t\to 0}\frac{2\ln(1+t)+1-\sqrt{1+4t}}{t^2} = \Bigg/ \begin{array}{l} (1+s)^{1/2} = 1+\frac{1}{2}s+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}s^2+O(s^3) \\ (1+4t)^{1/2} = 1+2t-\frac{1}{8}\cdot 16t^2+O(t^3) \\ \ln(1+t) = t-\frac{t^2}{2}+O(t^3) \end{array} \Bigg/ =$$

$$= \lim_{t\to 0}\frac{2t-t^2+O(t^3)+\cancel{1}-(\cancel{1}+2t-2t^2+O(t^3))}{t^2} =$$

$$= \lim_{t\to 0}\frac{-t^2+2t^2+O(t^3)}{t^2} = \lim_{t\to 0}\frac{t^2(1+O(t))}{t^2} = 1.$$

5c

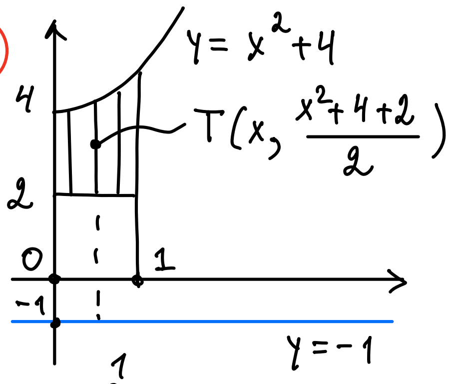
$$\lim_{x\to 3}\frac{\cos(2x-6)-1}{\sqrt[3]{x^2-6x+10}-1}=\left|\begin{array}{l} t=x-3\to 0 \\ \text{då } x\to 3 \\ x^2-6x+10= \\ (x-3)^2+1\end{array}\right|=\lim_{t\to 0}\frac{\cos 2t-1}{(1+t^2)^{1/3}-1}=$$

$$=\left|\begin{array}{l}(1+s)^{1/3}=1+\frac{1}{3}s+O(s^2)\\ \cos s=1-\frac{s^2}{2}+O(s^4)\end{array}\right|=\lim_{t\to 0}\frac{\cancel{1}-\frac{4t^2}{2}+O(t^4)-\cancel{1}}{\cancel{1}+\frac{1}{3}t^2+O(t^4)-\cancel{1}}=$$

$$=\lim_{t\to 0}\frac{-2t^2+O(t^4)}{\frac{1}{3}t^2+O(t^4)}=\lim_{t\to 0}\frac{\cancel{t^2}\left(-2+O(t^2)\right)}{\cancel{t^2}\left(\frac{1}{3}+O(t^2)\right)}=\frac{-2}{\frac{1}{3}}=-6.$$

Svar: a) 2 b) 1 c) -6.

6



$dV = 2\pi r\cdot \text{arean}$ där $r=\frac{x^2+6}{2}+1$

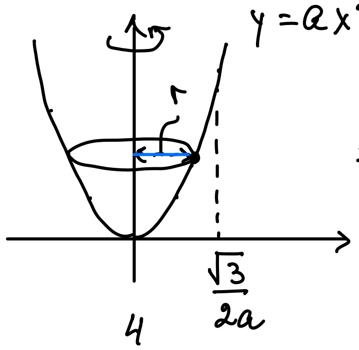
och $\text{arean}=(x^2+4-2)\,dx \Rightarrow$

$$V=\int_0^1 dV=2\pi\int_0^1\left(\frac{x^2}{2}+4\right)(x^2+2)\,dx=$$

$$=\pi\int_0^1(x^4+10x^2+16)\,dx=\pi\left(\frac{x^5}{5}+\frac{10}{3}x^3+16x\right)\Big|_0^1=\frac{293}{15}\pi\ (\text{v.e.})$$

Svar: $19\frac{8}{15}$ v.e.

7



$y=ax^2$ $dA=2\pi r\,ds$ där $r=x$, $ds=\sqrt{1+\left((ax^2)'\right)^2}\,dx$

$$\Rightarrow A=\int_0^{3/2a}dA=2\pi\int_0^{\sqrt{3}/2a}x\sqrt{1+4a^2x^2}\,dx=\left|\begin{array}{l}t=1+4a^2x^2\\ dt=8a^2x\,dx\\ x=0\Rightarrow\alpha=1\\ x=\frac{\sqrt{3}}{2a}\Rightarrow\beta=4\end{array}\right|=$$

$$=\frac{2\pi}{8a^2}\int_1^4\sqrt{t}\,dt=\frac{\pi}{4a^2}\cdot\left[\frac{t^{3/2}}{\frac{3}{2}}\right]_1^4=\frac{\pi}{4a^2}\cdot\frac{2}{3}(8-1)=\frac{\pi}{6a^2}\cdot 7$$

$$A=42\pi \iff 42\pi=\frac{\pi}{6a^2}\cdot 7\iff 6=\frac{1}{6a^2}\iff a^2=\frac{1}{6^2}\iff a=\pm\frac{1}{6}.$$

Svar: $a=\frac{1}{6}$ eller $a=-\frac{1}{6}$.