Lösningsskiss till tentamen i matematisk analys, del 2
764G07, TEN2, 2019-04-23

1a $\int \frac{\cos x}{2+\sin x}dx \Big/ \begin{matrix} t=2+\sin x \\ dt = \cos x\,dx \end{matrix} \Big/ = \int \frac{dt}{t} = \ln|t| + C =$

$= \ln|2+\sin x| + C$

1b $\int \ln\sqrt{x}\,dx = \frac{1}{2}\int \ln x\,dx \Big/ \begin{matrix} PI \\ g=\ln x \Rightarrow g' = \frac{1}{x} \\ f=1 \Rightarrow F = x \end{matrix} \Big/ = \frac{1}{2}\left(x\ln x - \int \frac{x}{x}dx\right)$

$= \frac{1}{2}x\ln x - \frac{x}{2} + C$

1c $\int \frac{3x^2+7}{x^3-x^2+4x-4}dx = \Big/ \frac{3x^2+7}{(x^2+4)(x-1)} = \frac{Ax+B}{x^2+4} + \frac{C}{x-1} = \frac{x+1}{x^2+4} + \frac{2}{x-1} \Big/$

$= \int \frac{x}{x^2+4}dx + \int \frac{dx}{x^2+4} + 2\int \frac{dx}{x-1} = \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\arctan\frac{x}{2} + 2\ln|x-1| + C$

Svar: a) $\ln|2+\sin x| + C$ b) $\frac{1}{2}x\ln x - \frac{x}{2} + C$ c) $\frac{1}{2}\ln(x^2+4) + 2\ln|x-1| + \frac{1}{2}\arctan\frac{x}{2} + C$

2. $y' - y\frac{x}{\sqrt{1+x^2}} = xe^{\sqrt{1+x^2}}$ $IF = e^{-\int \frac{x}{\sqrt{1+x^2}}dx} = e^{-\sqrt{1+x^2}}$

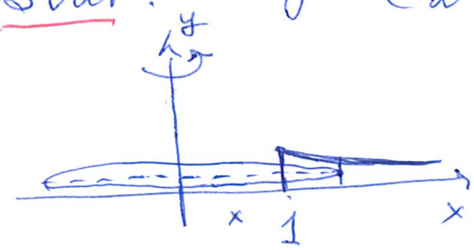
$y'e^{-\sqrt{1+x^2}} - y\frac{x}{\sqrt{1+x^2}}e^{-\sqrt{1+x^2}} = x \Leftrightarrow \left(ye^{-\sqrt{1+x^2}}\right)' = x \Leftrightarrow$

$ye^{-\sqrt{1+x^2}} = \frac{x^2}{2} + C \Leftrightarrow y = \left(\frac{x^2}{2} + C\right)e^{\sqrt{1+x^2}}$

$y(0) = 3 \Rightarrow 3 = C\cdot e \Rightarrow C = 3e^{-1}$

Svar: $y = \left(\frac{x^2}{2} + \frac{3}{e}\right)e^{\sqrt{1+x^2}}$.

3



$dV = 2\pi x\,dA = 2\pi x f(x)dx$

$V = 2\pi\int_1^\infty xe^{-2x}dx = 2\pi \lim_{M\to\infty}\int_1^M xe^{-2x}dx$

$\int_1^M xe^{-2x}dx \Big/ \begin{matrix} PI \\ g = x \quad g' = 1 \\ f = e^{-2x} \quad F = -\frac{1}{2}e^{-2x} \end{matrix} \Big/ = \left(-\frac{x}{2}e^{-2x}\right)_1^M + \int_1^M \frac{1}{2}e^{-2x}dx =$

$= -\frac{M}{2}e^{-2M} + \frac{1}{2}e^{-2} - \left(\frac{1}{4}e^{-2x}\right)_1^M = -\frac{M}{2}e^{-2M} + \frac{1}{2}e^{-2} - \frac{1}{4}e^{-2M} + \frac{1}{4}e^{-2}$

$\Rightarrow V = 2\pi \lim_{M\to\infty}\left(\frac{3}{4}e^{-2} - \frac{M}{2}e^{-2M} - \frac{1}{4}e^{-2M}\right) = 2\pi\frac{3}{4}e^{-2}$ (v.e.)

Svar: $V = \frac{3\pi}{2}e^{-2}$ v.e.

4a $\lim_{x\to 0} \frac{\sin(3x) - 3\ln(1+x)}{5x(1-e^{2x})}$ $\Big/$ $e^{2x} = 1+2x+O(x^2)$, $\sin 3x = 3x + O(x^3)$, $\ln(1+x) = x - \frac{x^2}{2} + O(x^3)$ $\Big/$ $=$

$= \lim_{x\to 0} \frac{\cancel{3x} + O(x^3) - \cancel{3x} + \frac{3}{2}x^2 + O(x^3)}{5x(\cancel{1} - \cancel{1} - 2x - O(x^2))} = \lim_{x\to 0} \frac{\frac{3}{2}x^2 + O(x^3)}{-10x^2 + O(x^3)} =$

$= -\frac{3}{20}$

4b $\lim_{x\to -2} \frac{\cos(x+2) - 1}{(x+3)^{2/3} - 1 - \frac{2}{3}(x+2)}$ $\Big/$ $t = x+2$, $t\to 0$ då $x \to -2$ $\Big/$ $=$

$= \lim_{t\to 0} \frac{\cos t - 1}{(1+t)^{2/3} - 1 - \frac{2}{3}t} = \lim_{t\to 0} \frac{\cancel{1} - \frac{t^2}{2} + O(t^4) - \cancel{1}}{\cancel{1} + \cancel{\frac{2}{3}t} + \frac{\frac{2}{3}(\frac{2}{3}-1)}{2} t^2 + O(t^3) \cancel{-1} \cancel{-\frac{2}{3}t}} =$

$= \lim_{t\to 0} \frac{-\frac{t^2}{2} + O(t^4)}{-\frac{1}{9}t^2 + O(t^3)} = \lim_{t\to 0} \frac{t^2(-\frac{1}{2} + O(t^2))}{t^2(-\frac{1}{9} + O(t))} = \frac{9}{2}$

Svar: a) $-\frac{3}{20}$ b) $\frac{9}{2}$

5. $f(x) = \cos x - \frac{1}{2}e^{-x^2} = 1 - \cancel{\frac{x^2}{2}} + \frac{x^4}{4!} + O(x^6) -$
$-\frac{1}{2}(1 \cancel{- x^2} + \frac{x^4}{2} + O(x^6)) = 1 - \frac{5}{4!}x^4 + O(x^6)$

$f(0) = 1 \Rightarrow f(x) - f(0) = -\frac{5}{4!}x^4 + O(x^6) < 0$ för x nära 0 $\Rightarrow$ $f(x) < f(0)$ för alla x nära 0

Svar: $x = 0$ är maximipunkt.

6. $x = \cos^2 t$, $y = \sin^2 t$, $0 \le t \le \frac{\pi}{2}$.

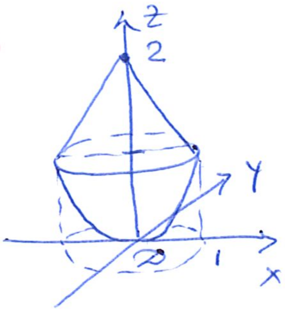
$s = \int_0^{\pi/2} \sqrt{x'^2 + y'^2}\, dt$ $\Big/$ $x' = -2\cos t \sin t = -\sin 2t$, $y' = 2\sin t \cos t = \sin 2t$ $\Big/$

$= \int_0^{\pi/2} \sqrt{\sin^2 2t + \sin^2 2t}\, dt = \sqrt{2} \int_0^{\pi/2} |\sin 2t|\, dt =$

$= \sqrt{2}\left(-\frac{\cos 2t}{2}\right)_0^{\pi/2} = \sqrt{2}$ (l.e.)

Svar: $s = \sqrt{2}$ l.e.

7.



$$\begin{cases} z = x^2+y^2 \\ z = 2-\sqrt{x^2+y^2} \end{cases} \iff \begin{cases} z = x^2+y^2 \geq 0 \\ z = 2-\sqrt{z} \end{cases} \iff z = 1 \text{ ty } z \geq 0$$

$$D = \{(x,y) : x^2+y^2 \leq 1\}$$

$$V = \iint_D \left(2-\sqrt{x^2+y^2} - (x^2+y^2)\right) dx\,dy \; \Bigg/ \begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \\ J = r \\ D': \ 0 \leq r \leq 1 \\ \quad\ \ 0 \leq \varphi \leq 2\pi \end{array} \Bigg/ =$$

$$= \iint_{D'} (2-r-r^2)\, r\, dr\, d\varphi =$$

$$= \int_0^{2\pi} \left(\int_0^1 (2-r-r^2)\, r\, dr \right) d\varphi = 2\pi \int_0^1 (2r-r^2-r^3)\, dr =$$

$$= 2\pi \left(r^2 - \frac{r^3}{3} - \frac{r^4}{4} \right)\Bigg|_0^1 = 2\pi \left(1-\frac{1}{3}-\frac{1}{4}\right) = \frac{5}{6}\pi \ (\text{v.e.})$$

Svar: $V = \frac{5}{6}\pi$ v.e.