

2.25 Bestäm  $(A+I)^{10}$  om  $A^2=A$ .

$$\begin{aligned}(A+I)^{10} &= \left((A+I)^2\right)^5 = \left((A+I)(A+I)\right)^5 = \left(\underbrace{A^2}_=A + \underbrace{AI}_=A + \underbrace{IA}_=A + \underbrace{I \cdot I}_=I\right)^5 \\&= (3A+I)^5 = \left((3A+I)^2\right)^2 \cdot (3A+I) = \left((3A+I)(3A+I)\right)^2 (3A+I) \\&= \left(\underbrace{9A^2}_=A + \underbrace{3AI}_=A + \underbrace{3IA}_=A + \underbrace{I \cdot I}_=I\right)^2 (3A+I) = (15A+I)^2 (3A+I) \\&= (15A+I)(15A+I)(3A+I) = \left(\underbrace{225A^2}_=A + \underbrace{15AI}_=A + \underbrace{15IA}_=A + \underbrace{I \cdot I}\right) (3A+I) \\&= (255A+I)(3A+I) = \left(\underbrace{765A^2}_=A + \underbrace{255AI}_=A + \underbrace{3IA}_=A + \underbrace{I \cdot I}\right) = 1023A+I\end{aligned}$$

svor:  $(A+I)^{10} = 1023A+I$  då  $A^2=A$ .