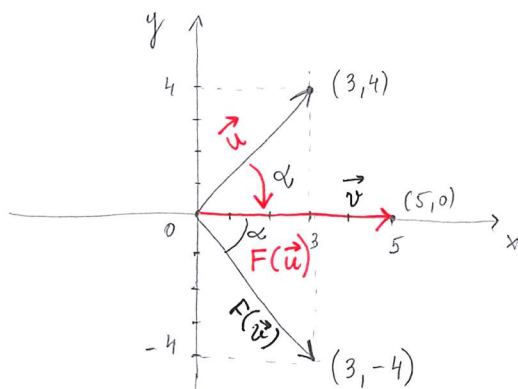


2.37 boken



$$\vec{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \text{ och } F(\vec{u}) = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \text{ vridning medurs}$$

$$\text{där } |\vec{u}| = \sqrt{9+16} = 5$$

$$\text{och } |F(\vec{u})| = \sqrt{25+0} = 5$$

det betyder (ser bilden) att om $\vec{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ blir $F(\vec{v}) = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$
(obs! vi vridit vinkeln α medurs vilket ger $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$)

$$\text{Alltså } \begin{cases} F(\vec{u}) = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \\ F(\vec{v}) = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} F\left(\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \\ F\left(\begin{pmatrix} 5 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} F(3\vec{e}_1 + 4\vec{e}_2) = 5\vec{e}_1 \\ F(5\vec{e}_1) = 3\vec{e}_1 - 4\vec{e}_2 \end{cases} \Leftrightarrow (\text{linearitetsegenskaperna ger då})$$

$$\Leftrightarrow \begin{cases} 3F(\vec{e}_1) + 4F(\vec{e}_2) = 5\vec{e}_1 & 1 \cdot (-3) & 1) \\ 5(F(\vec{e}_1)) = 3\vec{e}_1 - 4\vec{e}_2 & 1 \cdot (3) & 2) \end{cases} \Leftrightarrow \begin{cases} -15F(\vec{e}_1) - 20F(\vec{e}_2) = -25\vec{e}_1 \\ 15F(\vec{e}_1) & & = 9\vec{e}_1 - 12\vec{e}_2 \end{cases}$$

$$1) + 2) \text{ ger } -20F(\vec{e}_2) = -16\vec{e}_1 - 12\vec{e}_2 \Leftrightarrow F(\vec{e}_2) = \frac{4}{5}\vec{e}_1 + \frac{3}{5}\vec{e}_2 = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix} \underline{e}$$

$$\text{dvs 2) ger } 5F(\vec{e}_1) = 3\vec{e}_1 - 4\vec{e}_2 \Leftrightarrow F(\vec{e}_1) = \frac{3}{5}\vec{e}_1 - \frac{4}{5}\vec{e}_2 = \begin{pmatrix} 3/5 \\ -4/5 \end{pmatrix} \underline{e}$$

$$\text{Avbildningsmatris } A = \begin{bmatrix} F(\vec{e}_1) & F(\vec{e}_2) \end{bmatrix} = \begin{bmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$

$$\text{var: } A = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$