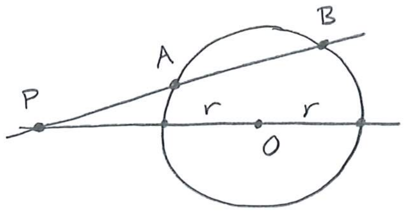
Lösningar, 91MA12, 92MA12, 2021-01-04

1.



$PA = 4$, $PB = 7$, $PO = 6$.

Dra linjen PO.

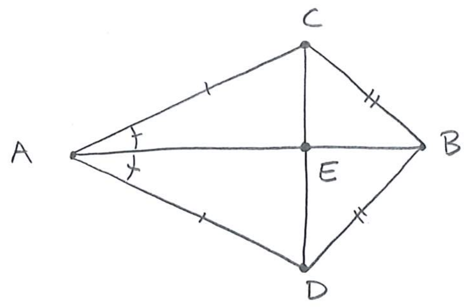
Kordasatsen ger:

$$PA \cdot PB = (PO - r)(PO + r),$$

$$4 \cdot 7 = (6-r)(6+r), \quad 28 = 36 - r^2, \quad r^2 = 8.$$

Svar: $\sqrt{8}$ cm.

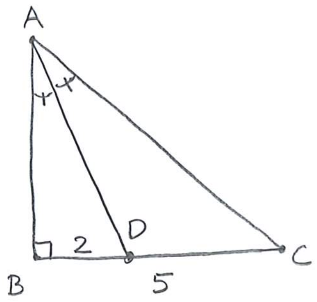
2.



Givet: $AC = AD$, $BC = BD$.

Vi har $\triangle ABC \cong \triangle ABD$ (SSS), så $\angle BAC = \angle BAD$. Detta ger $\triangle AEC \cong \triangle AED$ (SVS), så $EC = ED$, vilket skulle visas.

3.



Bisektrissatsen ger: $\dfrac{AB}{AC} = \dfrac{2}{5-2}$,

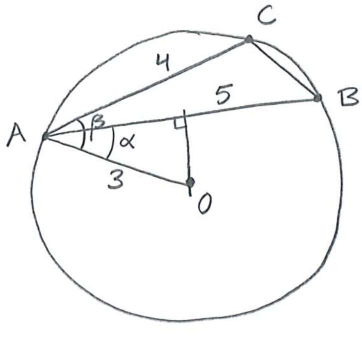
så $AB = \dfrac{2AC}{3}$.

Eftersom $AB^2 + BC^2 = AC^2$ (Pythagoras sats) får vi:

$$\frac{4AC^2}{9} + 25 = AC^2, \quad \frac{5AC^2}{9} = 25, \quad AC^2 = 9 \cdot 5.$$

Svar: $3\sqrt{5}$ cm.

4.



Låt $\alpha = \angle OAB$, $\beta = \angle OAC$.
Mittpunktsnormalen till AB går genom O, så $\cos\alpha = \frac{5/2}{3} = \frac{5}{6}$.
P.s.s. fås $\cos\beta = \frac{4/2}{3} = \frac{2}{3}$.

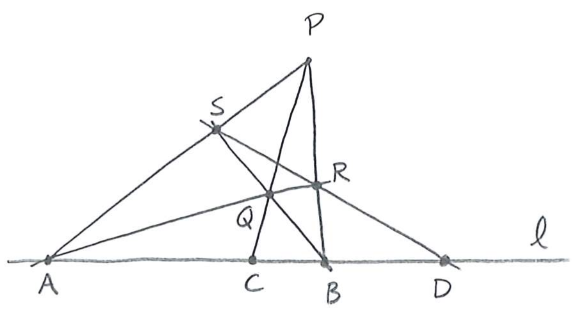
Detta ger $\sin\alpha = +\sqrt{1-\frac{5^2}{6^2}} = \frac{\sqrt{11}}{6}$, och $\sin\beta = +\sqrt{1-\frac{2^2}{3^2}} = \frac{\sqrt{5}}{3}$, så $\cos(\beta-\alpha) = \cos\beta\cos\alpha + {}$
${}+\sin\beta\sin\alpha = \frac{2}{3}\cdot\frac{5}{6} + \frac{\sqrt{5}}{3}\cdot\frac{\sqrt{11}}{6} = \frac{10+\sqrt{55}}{18}$. Nu ger cosinussatsen att $BC^2 = 4^2+5^2-2\cdot4\cdot5\cos(\beta-\alpha) =$

$$= 16+25-2\cdot4\cdot5\,\frac{10+\sqrt{55}}{18} = \frac{169-20\sqrt{55}}{9}.$$

<u>Svar:</u> $\frac{\sqrt{169-20\sqrt{55}}}{3}$ cm.

5. Se kompendiet.

6.



Cevas sats ger att

$$\frac{AC}{CB}\cdot\frac{BR}{RP}\cdot\frac{PS}{SA} = 1, \text{ och}$$

Menelaos sats ger att

$$\frac{AD}{DB}\cdot\frac{BR}{RP}\cdot\frac{PS}{SA} = 1.$$

Alltså är $\frac{AC}{CB} = \frac{AD}{DB}$, dvs $\underline{\underline{\frac{AC}{BC} = \frac{AD}{BD}}}$.