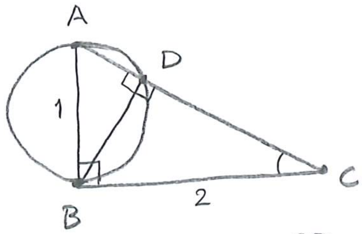
Lösningar, 91MA12, 92MA12, 2022-01-05

1.



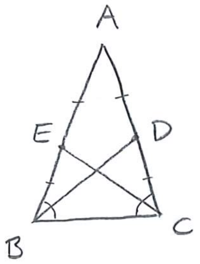
$AC = \sqrt{1^2+2^2} = \sqrt{5}$ (Pythagoras sats).

$\angle ADB = 90^\circ$ enl. periferivinkelsatsen.

$\triangle CDB \sim \triangle CBA$ (VV), så

$\frac{CD}{CB} = \frac{CB}{CA}$, $CD = \frac{2 \cdot 2}{\sqrt{5}}$. Svar: $\frac{4}{\sqrt{5}}$ cm.

2.



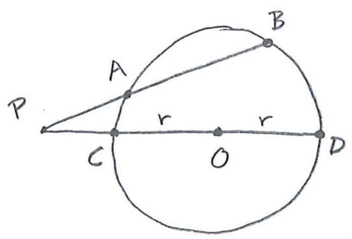
Givet: $AB = AC$.

BD och CE medianer, dvs $BE = EA$ och $CD = DA$, så $BE = \frac{AB}{2} = \frac{AC}{2} = CD$.

Basvinkelsatsen ger att $\angle ABC = \angle ACB$.

$\triangle EBC \cong \triangle DCB$ (SVS), så $BD = CE$.

3.



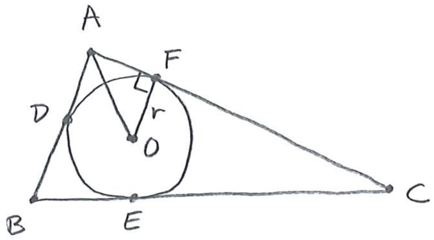
$PA = 2$, $PB = 5$, $PO = 4$.

Kordasatsen ger $PA \cdot PB = PC \cdot PD$,

så $2 \cdot 5 = (4-r)(4+r)$, $10 = 16 - r^2$,

så $r^2 = 6$. Svar: $r = \sqrt{6}$ cm.

4.



$AB = 3$, $AC = 5$, $BC = 6$.

$r = \frac{T}{p}$ där T = triangelns area,

och $p = \frac{AB + AC + BC}{2} = 7$.

Herons formel ger $T = \sqrt{7(7-3)(7-5)(7-6)} = \sqrt{7 \cdot 8}$,

så $r = \frac{\sqrt{7 \cdot 8}}{7} = \sqrt{\frac{8}{7}}$. $AF = p - BC$, ty:

$2AF = AF + AD = AC - CF + AB - BD = AB + AC - (CE + BE) =$

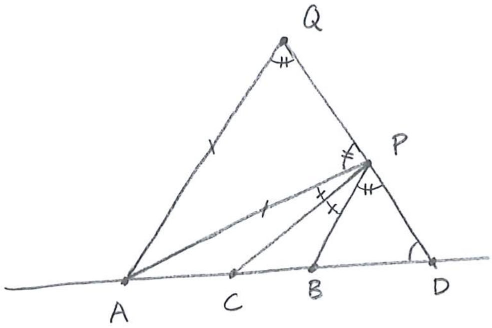
$= AB + AC - BC = 2p - 2BC$, så $AF = 7 - 6 = 1$.

Detta ger $AO^2 = 1^2 + \sqrt{\frac{8}{7}}^2 = \frac{15}{7}$.

Svar: $\sqrt{\frac{15}{7}}$ cm.

5. Se kompendiet.

6.



Dra linje genom A parallell med BP och låt Q vara skärningspunkten med linjen DP.

$\angle APQ = 90° - \angle APC = 90° - \angle BPC = \angle BPD$.

$\angle AQP = \angle BPD$ (alt.v.s.), så $AP = AQ$ (basv.s.).

Så $\frac{AC}{BC} = /\text{bis.s.}/ = \frac{AP}{BP} = \frac{AQ}{BP} = /\triangle AQD \sim \triangle BPD \text{ (vv)}/ = \frac{AD}{BD}$.