$$\alpha(h,k) = \frac{f(a+h, b+k) - f(a,b) - f_x'(a,b)\cdot h - f_y'(a,b)\cdot k}{\sqrt{h^2+k^2}}$$

$$\alpha(h,k)\cdot\sqrt{h^2+k^2} = f(a+h,b+k) - \underbrace{f(a,b) - f_x'(a,b)\cdot h - f_y'(a,b)k}$$

$$f(a+h,b+k) = f(a,b) + \underbrace{f_x'(a,b)h}_{\downarrow\, 0} + \underbrace{f_y'(a,b)k}_{\downarrow\, 0} + \underbrace{\alpha(h,k)\cdot\sqrt{h^2+k^2}}_{\downarrow\,\downarrow\, 0}$$

$$|f(a+h, b+k) - f(a,b)| \leq F > 0$$

$$\Updownarrow$$

$$-F \leq f(a+h,b+k) - f(a,b) \leq F$$

$$\Updownarrow$$

$$f(a,b) - F \leq f(a+h,b+k) \leq f(a,b) + F$$

d. v. s.

$$f(a+h, b+k) \in [f(a,b)-F,\ f(a,b)+F]$$

$$\begin{tikzcd} (a,b) \subset \mathbb{R}_t \arrow[r, "g"] \arrow[rr, bend right] & \mathbb{R}^2_{(u,v)} \arrow[r, "f"] & \mathbb{R} \end{tikzcd}$$

$$f \circ g = h$$

$$t \longrightarrow (g_1(t), g_2(t)) \longrightarrow f(g_1(t), g_2(t))$$

$$\shortparallel$$

$$h(t)$$

$h'(t) = ?$

Obs

$$\frac{df}{dt} = \frac{\partial f}{\partial u} \cdot \frac{du}{dt} + \frac{\partial f}{\partial v} \cdot \frac{dv}{dt}$$

$$df = \frac{\partial f}{\partial u} \cdot du + \frac{\partial f}{\partial v} \cdot dv$$

$$\begin{tikzcd} \underset{(x,y)}{R^2} \arrow[r, "g"] \arrow[rr, bend right, "f \circ g"'] & \underset{(u,v)}{R^2} \arrow[r, "f"] & R \end{tikzcd}$$

$$\cdot (x,y) \xrightarrow{g} (u(x,y), v(x,y)) \xrightarrow{f} \cdot f(u(x,y), v(x,y))$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \parallel$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad h(x,y)$$

$$\frac{\partial h}{\partial x}, \quad \frac{\partial h}{\partial y} = ?$$

$$\left[\begin{array}{l} \dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial u} \cdot \dfrac{\partial u}{\partial x} + \dfrac{\partial f}{\partial v} \cdot \dfrac{\partial v}{\partial x} \\[2ex] \dfrac{\partial f}{\partial y} = \dfrac{\partial f}{\partial u} \cdot \dfrac{\partial u}{\partial y} + \dfrac{\partial f}{\partial v} \cdot \dfrac{\partial v}{\partial y} \end{array}\right.$$