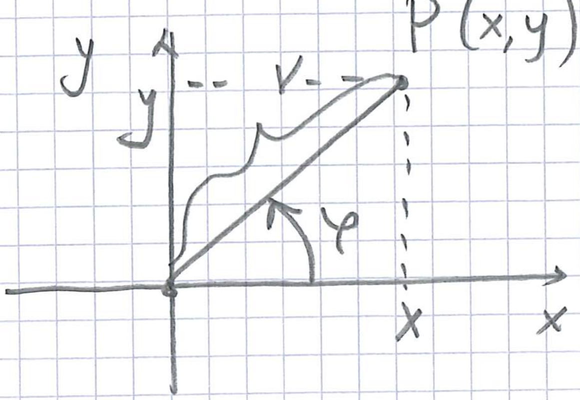
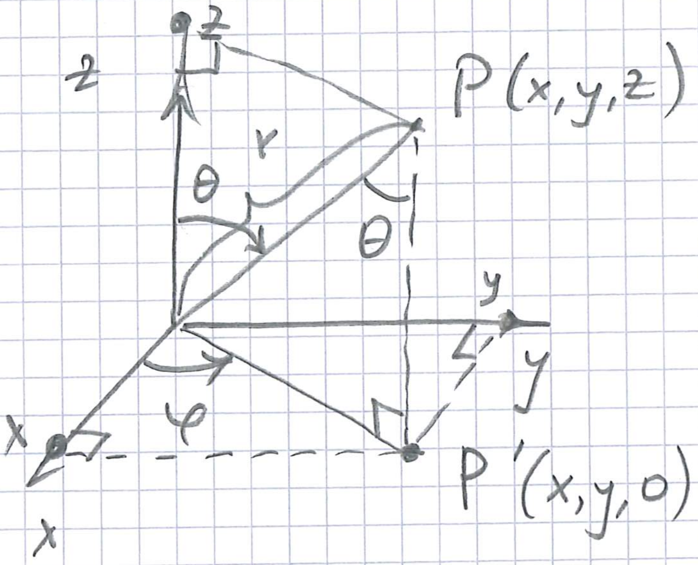
Ex.1 (Polära koordinater)



$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$r > 0$, $x^2 + y^2 = r^2$

Extra uppgift: (Sfäriska koordinater)



$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

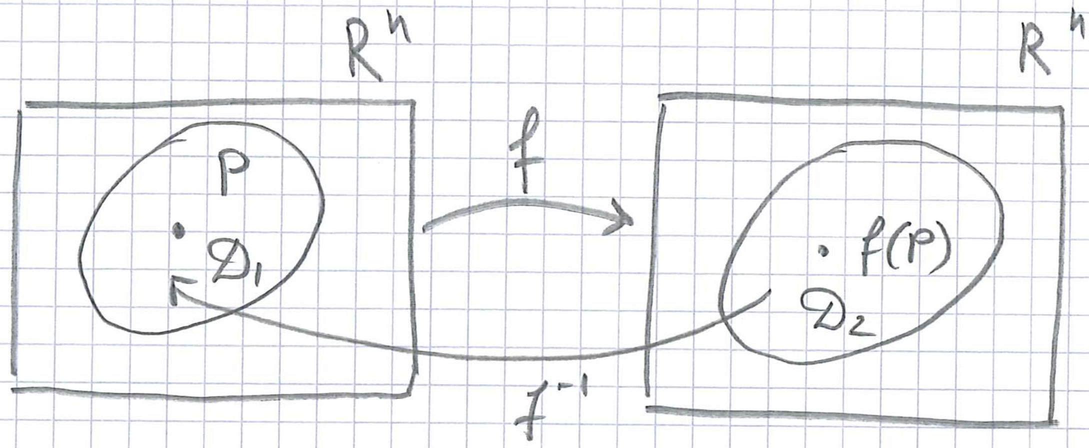
$z = r\cos\theta$

$x^2 + y^2 + z^2 = r^2$

<u>Sats</u> $f: \mathcal{D} \subseteq R^n \to R^n$, $n \geq 1$, <u>o</u> $P \in \mathcal{D}$

Om $\dfrac{\partial(f_1 \dots f_n)}{\partial(x_1 \dots x_n)}(P) \neq 0$ så är

f <u>lokalt inverterbar</u> nära P.



$\exists$ öppna delmängder till R^n <u>o</u>

$f^{-1}: \mathcal{D}_2 \subseteq R^n \to R^n$ s.a.

$P \in \mathcal{D}_1 \subseteq \mathcal{D}$, $\mathcal{D}_2 = f(\mathcal{D}_1)$, $f^{-1}(\mathcal{D}_2) = \mathcal{D}_1$ <u>o</u>

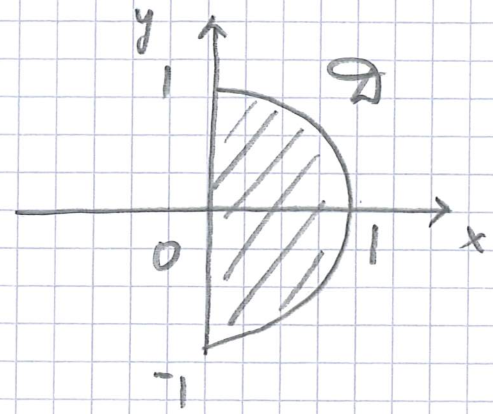
$f(f^{-1}(y)) = y \quad \forall y \in \mathcal{D}_2$

$f^{-1}(f(x)) = x \quad \forall x \in \mathcal{D}_1$

Ex 3.

$$I = \iint_{D_{xy}} x \, dx\, dy, \text{ där } D = \{x \geq 0,\ x^2 + y^2 \leq 1\}$$

Rita D:



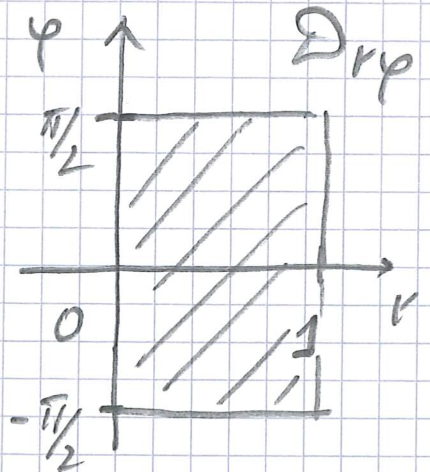
använd:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

Obs

$$\left.\begin{array}{l} 0 \leq r \leq 1 \\ -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2} \end{array}\right\} D_{r\varphi}$$

$$\Rightarrow I = \iint_{D_{r\varphi}} (r\cos\varphi) \cdot \overset{= r}{\left| \frac{\partial\, x\, y}{\partial\, r\, \varphi} \right|} \, dr\, d\varphi$$

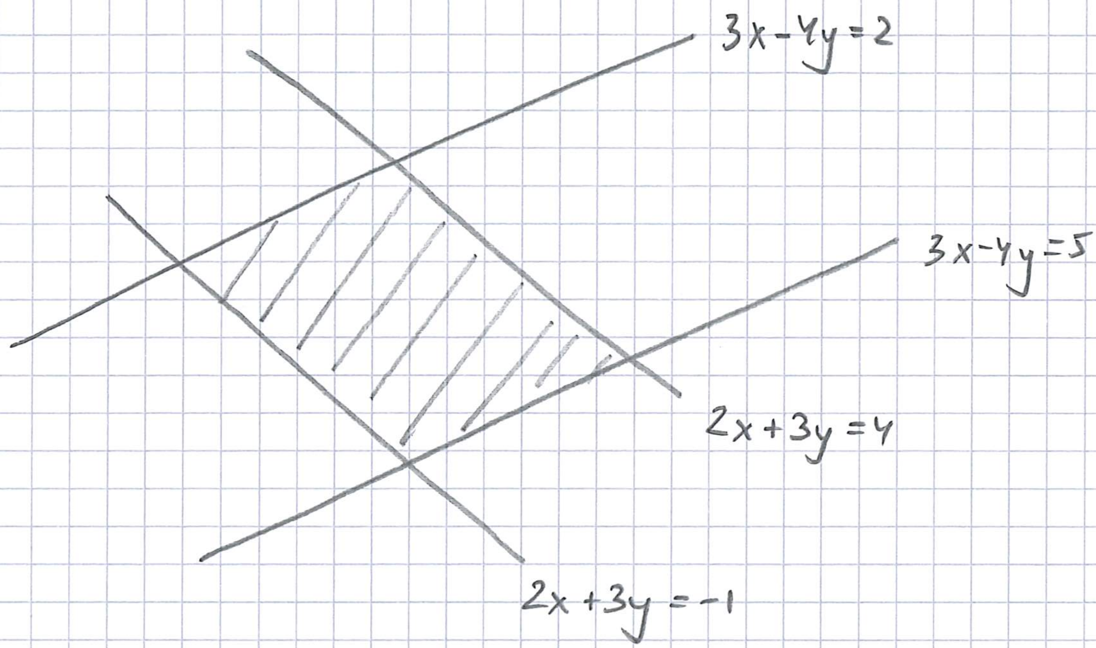


$$\Rightarrow I = \int_0^1 \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r^2 \cos\varphi \, d\varphi \right) dr =$$

$$\ldots = 2/3$$

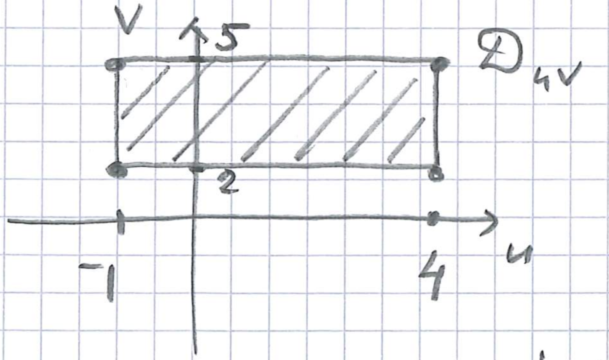
Ex 4 Finn arean A av

$$D_{xy} = \{-1 \le 2x+3y \le 4,\ 2 \le 3x-4y \le 5\}.$$



Obs $A = \iint_{D_{xy}} 1\cdot dx\,dy.$

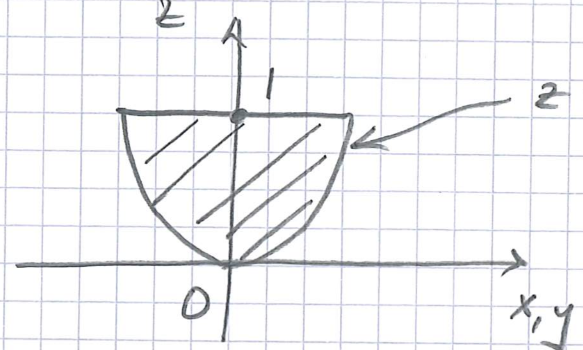
Använd substitutionen $\begin{cases} u = 2x+3y \\ v = 3x-4y \end{cases}$



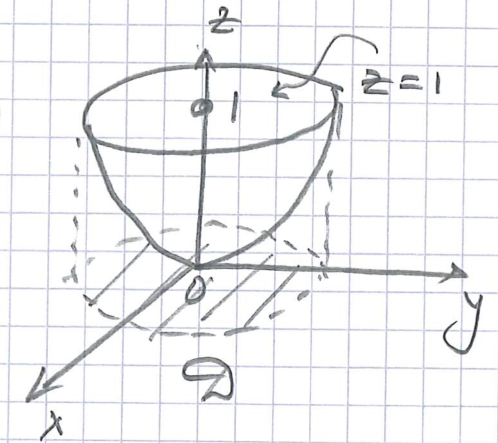
$$\Rightarrow A = \iint_{D_{uv}} 1\cdot \overbrace{\left|\frac{\partial xy}{\partial uv}\right|}^{\frac{1}{17}} du\,dv = \ldots = \frac{15}{17}$$

Ex 5 Beräkna volymen V av en tekopp av formen $z = x^2+y^2$, $0 \le z \le 1$

Bild:



eller



Obs $V = \iint_D (1-(x^2+y^2))\,dx\,dy$

där $D = \{x^2+y^2 \le 1\}$.

(projektionen av tekoppen på x,y-planet)

$V = \Big/$ polära koordinater $\Big/ = \int_0^1 \left(\int_0^{2\pi} (1-r^2)\,r\,d\varphi\right) dr =$

$\ldots = \pi/2$