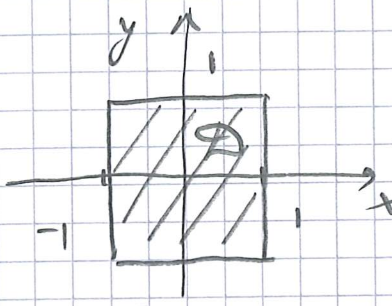
5.1 (a) $I = \iint_{D} (x+y)^2\,dx\,dy$, $D = \{|x|\le 1,\ |y|\le 1\}$



Obs $D = \{-1 \le x \le 1,\ -1 \le y \le 1\}$

$$\Rightarrow I = \int_{-1}^{1}\left(\int_{-1}^{1} (x+y)^2\,dy\right)dx =$$

$$= \int_{-1}^{1}\left(\frac{(x+y)^3}{3}\Big|_{-1}^{1}\right)dx = \frac{1}{3}\int_{-1}^{1}\left((x+1)^3-(x-1)^3\right)dx =$$

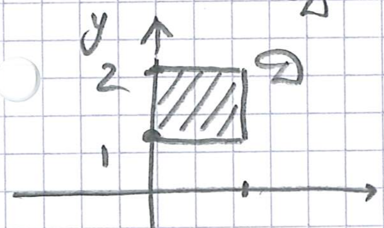
/ $a^3-b^3=(a-b)(a^2+ab+b^2)$ /

$$= \frac{1}{3}\int_{-1}^{1} 2\cdot\left((x+1)^2+(x+1)(x-1)+(x-1)^2\right)dx =$$

$$= \frac{2}{3}\int_{-1}^{1}(2x^2+2+x^2-1)\,dx = \frac{2}{3}\int_{-1}^{1}(3x^2+1)\,dx =$$

$$= \frac{2}{3}(x^3+x)\Big|_{-1}^{1} = \frac{2}{3}(2+2) = \frac{8}{3}$$

(b) $I = \iint_{D} e^{y^2}(3x^2-2x+y)\,dx\,dy$, $D = \{0 \le x \le 1,\ 1 \le y \le 2\}$



Obs $I = \int_{0}^{1}\left(\int_{1}^{2} e^{y^2}(3x^2-2x+y)\,dy\right)dx =$

$$= \underbrace{\int_{0}^{1}\left(\int_{1}^{2} e^{y^2}(3x^2-2x)\,dy\right)dx}_{=I_1} + \underbrace{\int_{0}^{1}\left(\int_{1}^{2} e^{y^2}\cdot y\,dy\right)dx}_{=I_2}$$

$$I_1 = \int_{0}^{1}(3x^2-2x)\underbrace{\left(\int_{1}^{2} e^{y^2}\,dy\right)}_{K\ \text{(konstant)}}dx = K\cdot\int_{0}^{1}(3x^2-2x)\,dx =$$

$$= K\cdot(x^3 - x^2)\Big|_0^1 = K(1-1-0) = K\cdot 0 = 0$$

$$I_2 = \left| \begin{matrix} t = y^2 \\ dt = 2y\,dy \end{matrix} \quad y\,dy = \frac{dt}{2} \right| = \int_0^1 \left(\int_{y=1}^{y=2} e^t \frac{dt}{2} \right) dx =$$

$$= \frac{1}{2}\int_0^1 \left(e^t \Big|_{y=1}^{y=2} \right) dx = \frac{1}{2}\int_0^1 \left(e^{y^2} \Big|_1^2 \right) dx =$$

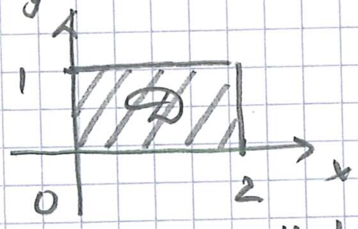
$$= \frac{1}{2}\int_0^1 (e^4 - e^1)\,dx = \frac{e^4 - e^1}{2}.$$

$$\Rightarrow \quad I = I_1 + I_2 = 0 + \frac{e^4 - e^1}{2} = \frac{e^4 - e^1}{2}$$

Obs Man kan testa också formeln:

$$I = \int_{-1}^{2} \left(\int_0^1 e^{y^2} (3x^2 - 2x + y)\,dx \right) dy.$$

(c) $I = \iint_{\mathcal{D}} x e^{xy}\,dx\,dy, \quad \mathcal{D} = \{0 \le x \le 2, \; 0 \le y \le 1\}.$



$$I = \int_0^2 \left(\int_0^1 x e^{xy}\,dy \right) dx = \left| \begin{matrix} t = xy \\ dt = x\,dy \end{matrix} \right|$$

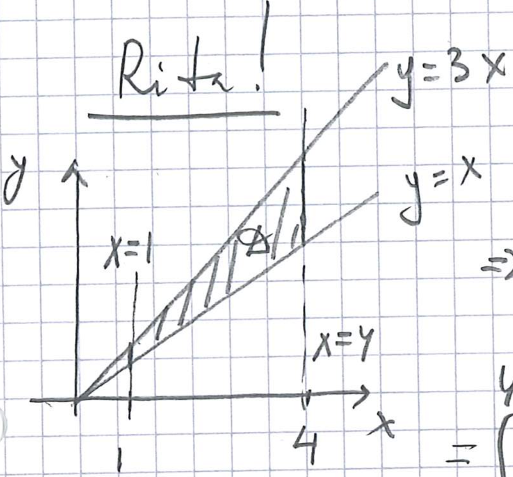
$$= \int_0^2 \left(\int_{y=0}^{y=1} e^t\,dt \right) dx = \int_0^2 e^t \Big|_{y=0}^{y=1} = \int_0^2 \left(e^{xy} \Big|_0^1 \right) dx =$$

$$= \int_0^2 (e^x - 1)\,dx = (e^x - x)\Big|_0^2 = e^2 - 2 - (1 - 0) =$$

$$= e^2 - 3$$

5.2 (b) $\quad I = \iint_{D} \frac{x}{y}\,dx\,dy$, D är ett område definierad av linjerna: $x=1,\ x=4,\ y=x,\ y=3x$

Rita!



$$\Rightarrow I = \int_1^4 \left(\int_x^{3x} \frac{x}{y}\,dy \right) dx =$$

$$= \int_1^4 x \left(\ln|y| \Big|_x^{3x} \right) dx = \left| \text{Obs } x>0 \right| =$$

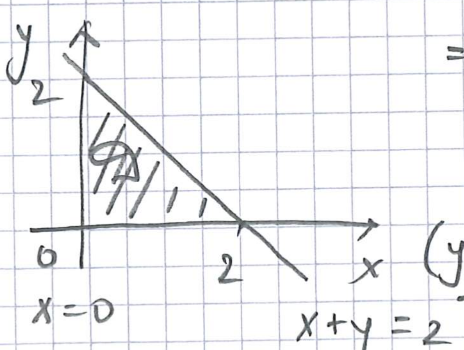
$$= \int_1^4 x(\ln 3x - \ln x)\,dx = \left| \begin{array}{l} \ln(ab) = \ln a + \ln b \\ \text{om } a,b>0 \end{array} \right| =$$

$$= \int_1^4 x(\ln 3 + \ln x - \ln x)\,dx = \ln 3 \int_1^4 x\,dx = \ln 3 \cdot \frac{x^2}{2}\Big|_1^4 =$$

$$= \frac{1}{2}\ln 3 \cdot (16-1) = \frac{15}{2}\ln 3$$

5.3 $\quad I = \iint_{D} (x+2y)\,dx\,dy$, D begränsas av linjerna: $x=0,\ y=0,\ x+y=2$

Rita!



$(y=0)$

$(y=2-x)$

$$\Rightarrow I = \int_0^2 \left(\int_0^{2-x} (x+2y)\,dy \right) dx =$$

$$= \int_0^2 \left(xy + y^2 \right)\Big|_0^{2-x} dx =$$

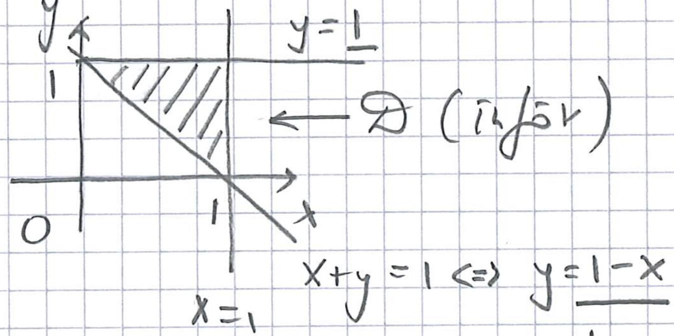
$$= \int_0^2 \left(x(2-x) + (2-x)^2 - 0 \right) dx =$$

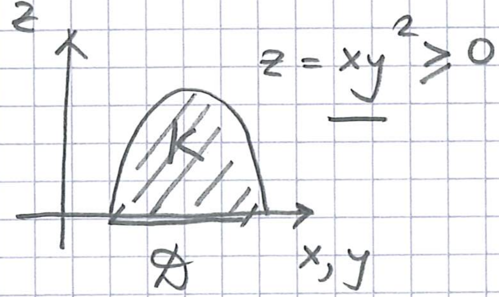
$$= \int_0^2 (2x - x^2 + x^2 - 4x + 4)\,dx = \int_0^2 (4 - 2x)\,dx =$$

$$= (4x - x^2)\Big|_0^2 = 8 - 4 - 0 = 4$$

5.4 Beräkna volymen av den kropp K som ligger under ytan $z = xy^2$ o ovanför triangelskivan $x \le 1,\ y \le 1,\ x + y \ge 1$ i x,y-planet.

Rita!





$$V = \iint_{\mathcal{D}} xy^2\,dx\,dy = \int_0^1 \left(\int_{1-x}^1 xy^2\,dy \right) dx =$$

$$= \int_0^1 \left(\frac{xy^3}{3} \right)\Bigg|_{1-x}^1 dx = \frac{1}{3} \int_0^1 x(1-x)^3\,dx =$$

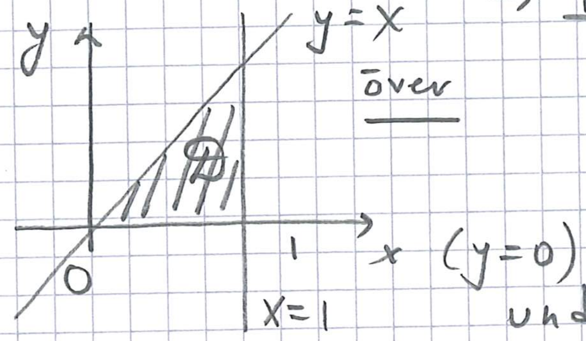
$$\frac{1}{3} \int_0^1 x(1 - 3x + 3x^2 - x^3)\,dx = \frac{1}{3} \int_0^1 (x - 3x^2 + 3x^3 - x^4)\,dx$$

$$= \frac{1}{3} \left(\frac{x^2}{2} - x^3 + \frac{3x^4}{4} - \frac{x^5}{5} \right)\Bigg|_0^1 = \frac{1}{3} \left(\frac{1}{2} - 1 + \frac{3}{4} - \frac{1}{5} \right) =$$

$$= \frac{1}{60}$$

5.5 (b) $\quad I = \iint_{\mathcal{D}} e^{-x-y}\,dx\,dy,\quad \mathcal{D} = \{y \geq 0,\ x \leq 1,\ y \leq x\}$

Rita!



över

$$\Rightarrow I = \int_0^1 \left(\int_0^x e^{-x-y}\,dy \right) dx =$$

under

$$= \int_0^1 \left(-e^{-x-y} \right)\Big|_0^x dx =$$

$$= -\int_0^1 \left(e^{-2x} - e^{-x} \right) dx = -\left(-\tfrac{1}{2} e^{-2x} + e^{-x} \right)\Big|_0^1 =$$

$$= -\left(\left(-\tfrac{1}{2} e^{-2} + e^{-1} \right) - \left(-\tfrac{1}{2} + 1 \right) \right) = \tfrac{1}{2} e^{-2} - e^{-1} + \tfrac{1}{2}$$

5.5 (c) $\quad I = \iint_{\mathcal{D}} \sqrt{x+y}\,dx\,dy,\quad \mathcal{D}$ är samma som ovan

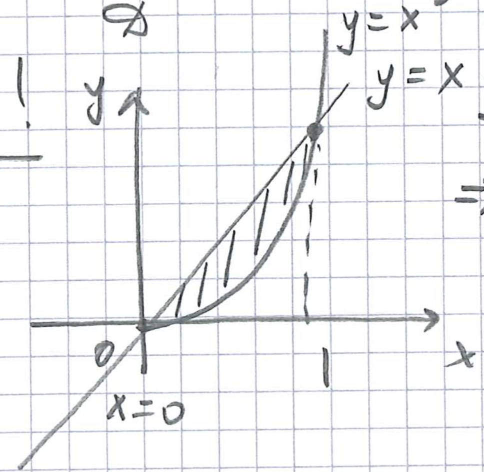
$$\Rightarrow I = \int_0^1 \left(\int_0^x \sqrt{x+y}\,dy \right) dx = \int_0^1 \frac{(x+y)^{3/2}}{3/2}\Big|_0^x dx =$$

$$= \frac{2}{3} \int_0^1 \left((2x)^{3/2} - x^{3/2} \right) dx = \frac{2}{3} \cdot \left(2^{3/2} - 1 \right) \int_0^1 x^{3/2}\,dx =$$

$$= \frac{2}{3} \left(2^{3/2} - 1 \right) \cdot \frac{x^{5/2}}{5/2}\Big|_0^1 = \frac{4}{15} \left(2^{3/2} - 1 \right)$$

$\underline{5.7}$ $I = \iint_{D} \sin(x^2)\,dx\,dy$, $D = \{ 0 \le x^3 \le y \le x \}$.

$\underline{\text{Rita!}}$



$$\Rightarrow I = \int_0^1 \left(\int_{x^3}^{x} \sin(x^2)\,dy \right) dx =$$

$$= \int_0^1 \sin x^2 \left(\int_{x^3}^{x} 1 \cdot dy \right) dx =$$

$$= \int_0^1 \sin(x^2) \cdot y \Big|_{x^3}^{x} dx = \int_0^1 (x - x^3) \sin(x^2)\,dx =$$

$$= \left| \begin{array}{ll} t = x^2 & x\,dx = \frac{dt}{2} \\ dt = 2x\,dx, & \end{array} \right| = \int_{x=0}^{x=1} (1-t) \sin t \cdot \frac{dt}{2} =$$

$$= \frac{1}{2} \int_{x=0}^{x=1} (1-t) \sin t\,dt = \dots$$

$$\Big/ \int \underset{\downarrow\,-1}{(1-t)} \overset{-\cos t\,\uparrow}{\sin t}\,dt \overset{p.i}{=} (1-t)(-\cos t) - \int (-1) \cdot (-\cos t)\,dt =$$

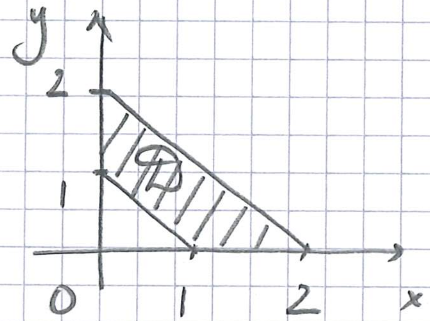
$$= (t-1)\cos t - \sin t + C \Big/ \Rightarrow$$

$$I = \frac{1}{2} \left((x^2 - 1) \cos x^2 - \sin x^2 \right) \Big|_0^1 =$$

$$= \frac{1}{2} \left((0 - \sin 1) - (-1 - 0) \right) = \underline{\frac{1 - \sin 1}{2}}$$

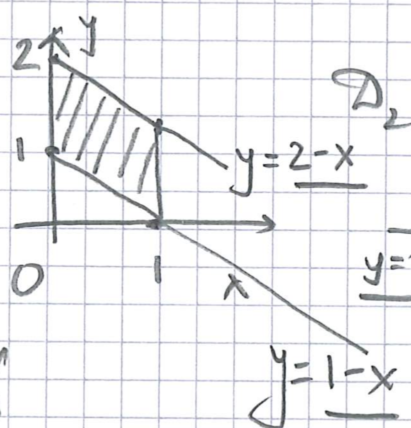
5.8 $I = \iint_{D} e^{x+y}\,dxdy$, D är fyrhörningen med hörn i $(1,0), (2,0), (0,1)$ o $(0,2)$

Rita!

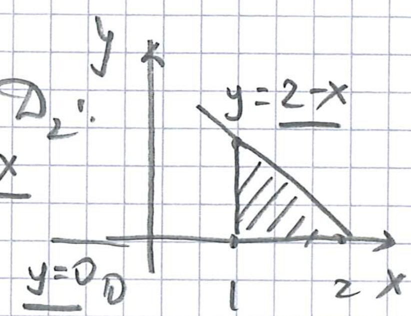


Dela upp D i två delar D_1 o D_2

D_1:



D_2:



$$\Rightarrow I = \iint_{D} = \underbrace{\iint_{D_1}}_{=I_1} + \underbrace{\iint_{D_2}}_{=I_2}$$

$$I_1 = \int_0^1 \left(\int_{1-x}^{2-x} e^{x+y}\,dy \right) dx = \int_0^1 e^{x+y} \Big|_{1-x}^{2-x} dx =$$

$$= \int_0^1 (e^2 - e^1)\,dx = e^2 - e^1$$

$$I_2 = \int_1^2 \left(\int_0^{2-x} e^{x+y}\,dy \right) dx = \int_1^2 e^{x+y} \Big|_0^{2-x} dx =$$

$$= \int_1^2 (e^2 - e^x)\,dx = (e^2 \cdot x - e^x) \Big|_1^2 = (e^2 \cdot 2 - e^2) - (e^2 \cdot 1 - e^1)$$

$$= \underline{e} \qquad \Rightarrow I = I_1 + I_2 = (e^2 - e) + e = e^2$$