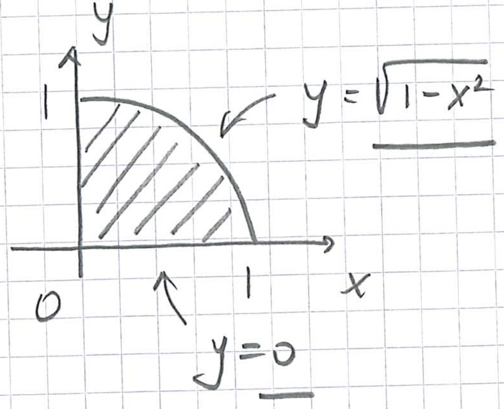
5.11 $\quad I = \iint_D \frac{xy}{(1+y^2)^2}\,dx\,dy, \quad D = \{ x \geq 0, y \geq 0, \; x^2+y^2 \leq 1 \}.$

Rita!



$$\Rightarrow I = \int_0^1 \left(\int_0^{\sqrt{1-x^2}} \frac{xy}{(1+y^2)^2}\,dy \right) dx =$$

$$= \int_0^1 x \left(\int_0^{\sqrt{1-x^2}} \frac{y}{(1+y^2)^2}\,dy \right) dx = \left| \begin{array}{l} t = 1+y^2, \quad y\,dy = \frac{dt}{2} \\ dt = 2y\,dy \end{array} \right| =$$

$$= \int_0^1 x \left(\int_{y=0}^{y=\sqrt{1-x^2}} \frac{1}{t^2} \cdot \frac{dt}{2} \right) = \frac{1}{2} \int_0^1 x \cdot \left(\frac{t^{-1}}{-1} \right) \Bigg|_{y=0}^{y=\sqrt{1-x^2}} dx =$$

$$= -\frac{1}{2} \int_0^1 x \cdot \frac{1}{t} \Bigg|_{y=0}^{y=\sqrt{1-x^2}} dx = -\frac{1}{2} \int_0^1 x \left[\frac{1}{1+y^2} \right]_0^{\sqrt{1-x^2}} dx =$$

$$= -\frac{1}{2} \int_0^1 x \cdot \left[\frac{1}{2-x^2} - 1 \right] dx = \frac{1}{2} \int_0^1 \frac{x\,dx}{x^2-2} + \frac{1}{2} \int_0^1 x\,dx =$$

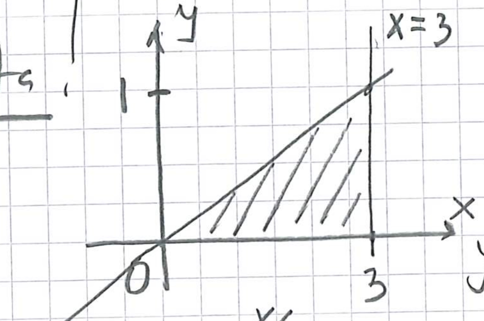
$$= \left| \begin{array}{l} t = x^2-2, \quad x\,dx = \frac{dt}{2} \\ dt = 2x\,dx \end{array} \right| = \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{t} \frac{dt}{2} + \frac{1}{2} \cdot \frac{x^2}{2} \Bigg|_0^1 =$$

$$= \frac{1}{4} \ln|t| \Bigg|_{x=0}^{x=1} + \frac{1}{4} = \frac{1}{4} \ln|x^2-2| \Bigg|_0^1 + \frac{1}{4} = \frac{1}{4} \underbrace{\ln 1}_{=0} - \frac{1}{4} \ln 2 + \frac{1}{4}$$

$$= \frac{1}{4}(1 - \ln 2)$$

5.12 (a) $I = \int_0^1 \left(\int_{3y}^3 e^{x^2} dx \right) dy$

Rita!



$x=3y \iff y = x/3$

Obs $\left\{ \begin{array}{l} 0 \leq y \leq 1 \\ 3y \leq x \leq 3 \end{array} \right\} = D$

$\Rightarrow I = \Big|$ Kasta om variablerna $\Big| =$

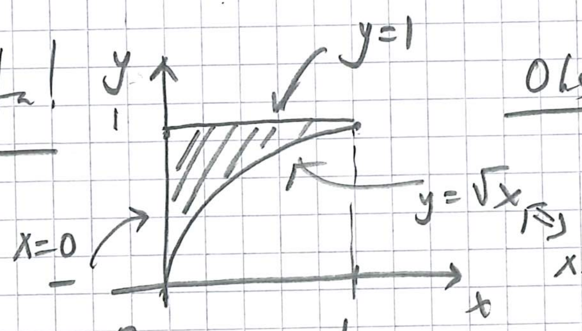
$= \int_0^3 \left(\int_0^{x/3} e^{x^2} dy \right) dx = \int_0^3 e^{x^2} \left(\int_0^{x/3} 1 \cdot dy \right) dx =$

$= \int_0^3 e^{x^2} \cdot y \Big|_0^{x/3} dx = \int_0^3 \frac{x}{3} e^{x^2} dx = \Big| \begin{array}{l} t = x^2, \quad x\,dx = \frac{dt}{2} \\ dt = 2x\,dx \end{array} \Big|$

$= \frac{1}{6} \int_{x=0}^{x=3} e^t dt = \frac{1}{6} e^t \Big|_{x=0}^{x=3} = \frac{1}{6} e^{x^2} \Big|_0^3 = \frac{1}{6}(e^9 - 1)$

(b) $\int_0^1 \left(\int_{\sqrt{x}}^1 e^{x/y} dy \right) dx$

Rita!



$y=\sqrt{x} \iff x = y^2$

Obs $D = \left\{ \begin{array}{l} 0 \leq x \leq 1 \\ \sqrt{x} \leq y \leq 1 \end{array} \right\}$

$I = \Big|$ Kasta om variablerna $\Big| = \int_0^1 \left(\int_0^{y^2} e^{x/y} dx \right) dy =$

$$= \left| \begin{array}{ll} t = x/y & dx = y\,dt \\ dt = \frac{1}{y}\,dx & \end{array} \right| = \int_0^1 \left(\int_{x=0}^{x=y^2} e^t y\,dt \right) dy =$$

$$= \int_0^1 y \left(\int_{x=0}^{x=y^2} e^t\,dt \right) dy = \int_0^1 y \left(e^t\right)\Big|_{x=0}^{x=y^2} dy =$$

e^y

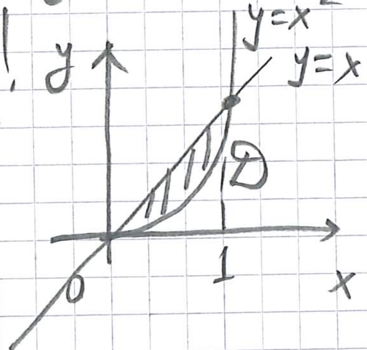
$$= \int_0^1 y \left(e^{x/y}\right)\Big|_0^{y^2} dy = \int_0^1 y\left(e^y - 1\right) dy = \int_0^1 y e^y\,dy - \int_0^1 y\,dy$$

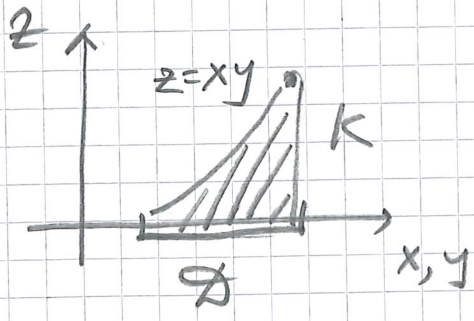
$$= y e^y \Big|_0^1 - \int_0^1 e^y\,dy - \frac{1}{2} = e - e^y\Big|_0^1 - \frac{1}{2} =$$

$$= e - (e-1) - \frac{1}{2} = \frac{1}{2}$$

5.13 Beräkna volymen av den kropp K som ligger mellan ytan $z = xy$ o x,y-planet o som begränsas av planet $y = x$ o ytan $y = x^2$

Rita!





Obs $x \cdot y > 0$ på D.

$$V = \iint_D xy\,dx\,dy = \int_0^1 \left(\int_{x^2}^{x} xy\,dy \right) dx =$$

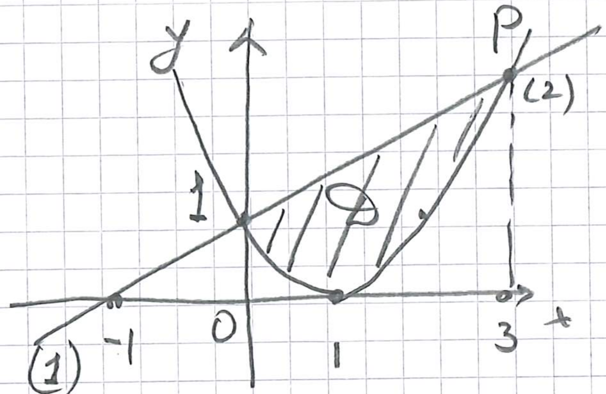
$$= \int_0^1 \frac{xy^2}{2}\Big|_{x^2}^{x} dx = \int_0^1 \frac{1}{2} \cdot x\left(x^2 - x^4\right) dx = \frac{1}{2}\int_0^1 \left(x^3 - x^5\right) dx =$$

$$= \frac{1}{2}\left(\frac{x^4}{4} - \frac{x^6}{6}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1}{4} - \frac{1}{6}\right) = \frac{1}{24}$$

5.15 $\mathcal{D}$ är begränsat av kurvorna

$y = x+1$ (1) o $y = x^2 - 2x + 1$, (2) Finn arean A av $\mathcal{D}$.

Rita!



Obs $x^2 - 2x + 1 = (x-1)^2$

Finn P:s x-koordinat! $x+1 = x^2 - 2x + 1$ eller

$x^2 - 3x = 0 \iff x_1 = 0,\ x_2 = \underline{3}$

$$A = \iint_{\mathcal{D}} 1\,dx\,dy = \left|\ \mathcal{D} = \{0 \le x \le 3,\ (x-1)^2 \le y \le x+1\}\ \right|$$

$$= \int_0^3 \left(\int_{(x-1)^2}^{x+1} 1\,dy \right) dx = \int_0^3 \left((x+1) - (x-1)^2\right) dx =$$

$$= \int_0^3 (-x^2 + 3x)\,dx = \left(-\frac{x^3}{3} + \frac{3x^2}{2}\right)\Big|_0^3 =$$

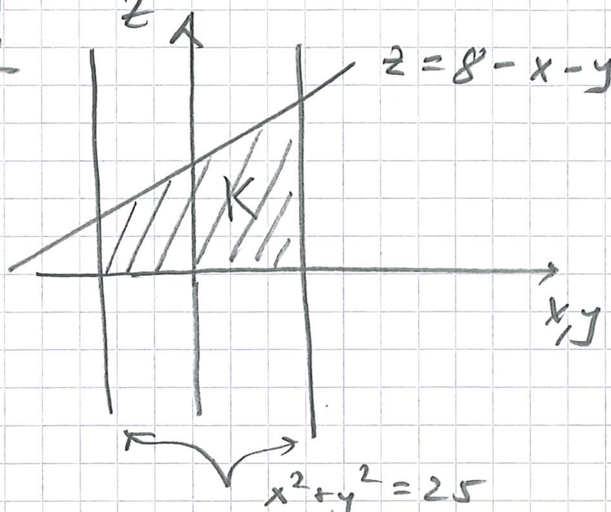
$$= -9 + \frac{27}{2} = \frac{9}{2}$$

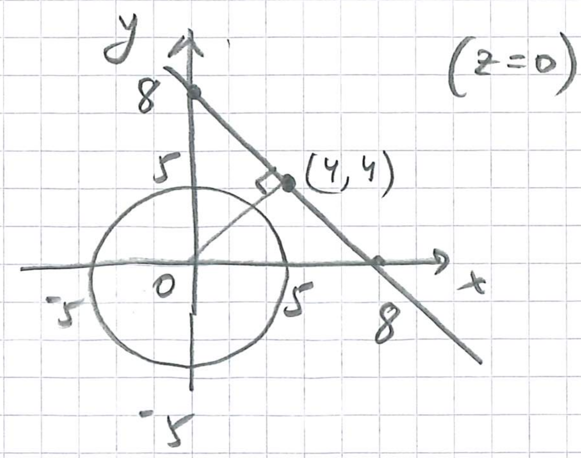
5.16 (b) Beräkna volymen av den kropp K som begränsas av

cylindern $x^2 + y^2 = 25$, planet $x + y + z = 8$ o

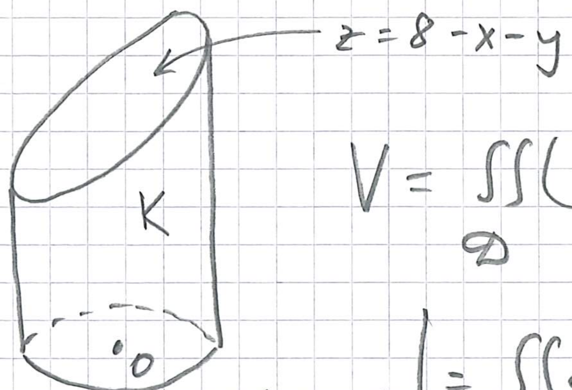
xy-planet.

Rita!





eller



$$V = \iint_{D} (8-x-y)\,dx\,dy =$$

$D = \{x^2+y^2 \le 25\}$

$$= \overset{=I_1}{\iint_{D} 8\,dx\,dy} - \overset{=I_2}{\iint_{D} x\,dx\,dy} - \underset{=I_3}{\iint_{D} y\,dx\,dy} = I_1 + I_2 + I_3$$

$$I_1 = 8 \underbrace{\iint_{D} 1\,dx\,dy}_{\text{arean av } D} = 8 \cdot \pi 5^2 = 200\pi$$

$$I_2 = \left| D = \left\{ \begin{array}{l} -5 \le x \le 5 \\ -\sqrt{25-x^2} \le y \le \sqrt{25-x^2} \end{array} \right\} \right| = \int_{-5}^{5} \left(\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} x\,dy \right) dx$$

$$= \int_{-5}^{5} \underbrace{x \cdot 2 \cdot \sqrt{25-x^2}}_{\text{udda}}\,dx = 0, \quad I_3 = 0 \text{ (på samma vis)}$$

$$\Rightarrow \quad V = 200\pi$$