5.19 Låt $f: R^m_{\bar{x}} \to R^n_{\bar{y}}$ vara en avbildning.

$$f: \begin{cases} y_1 = f_1(x_1,\dots,x_m) \\ \text{------------} \\ y_n = f_n(x_1,\dots,x_m) \end{cases}$$

Funktionalmatris: $\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_m} \\ \text{---} & \text{---} & \text{---} \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_m} \end{bmatrix} = FM.$

Om $n=m$ så är $\det(FM) = \frac{\partial(f_1,\dots,f_n)}{\partial(x_1,\dots,x_n)}$

funktionaldeterminant.

$$f: \begin{cases} y_1 = x_1 + x_2 + 2x_3 + 7 \\ y_2 = 3x_1 - 2x_2 + x_3 - 5 \\ y_3 = -x_1 - x_2 + 5x_3 - 4 \end{cases} \qquad FM = \begin{bmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{bmatrix}$$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| $

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad A$ (inför)

Är f inverterbar? Obs $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 7 \\ -5 \\ -4 \end{pmatrix}$

Räkna $\det A = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -2 & 1 \\ -1 & -1 & 5 \end{vmatrix} = 1\overset{-9}{\overset{\|}{(-10+1)}} - \overset{16}{\overset{\|}{(15+1)}} + 2\overset{-5}{\overset{\|}{(-3-2)}} =$

$= -9 - 16 - 10 \neq 0$

$\Rightarrow A^{-1}$ existerar $\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^{-1}\left(\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} - \begin{pmatrix} 7 \\ -5 \\ -4 \end{pmatrix}\right)$ Svar: ja.

5.20 $\begin{cases} y_1 = 3r\cos\theta \\ y_2 = 2r\sin\theta \end{cases}$ $f: (r,\theta) \longrightarrow (y_1, y_2)$

$$\Rightarrow \quad FM = \begin{bmatrix} 3\cos\theta & -3r\sin\theta \\ 2\sin\theta & 2r\cos\theta \end{bmatrix},$$

$$\frac{\partial(y_1,y_2)}{\partial(r,\theta)} = 6r(\cos^2\theta + \sin^2\theta) = 6r$$

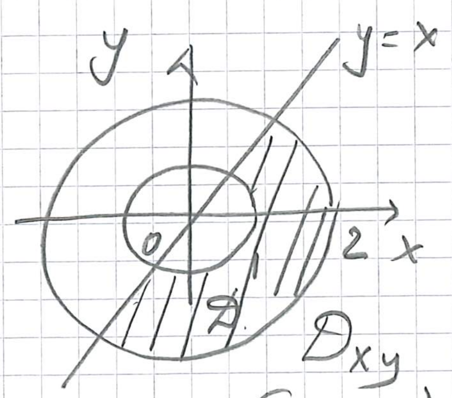
5.23 (a) $I = \iint_{D} x(x^2+y^2)\,dx\,dy$

$D = \{ 1 \le x^2+y^2 \le 4, \quad y \le x \}.$

Variabelbyte $I = \iint_{D_{xy}} f(x,y)\,dx\,dy = \left| \begin{matrix} x = x(u,v) \\ y = y(u,v) \end{matrix} \right|$

$$= \iint_{D_{uv}} f(x(u,v), y(u,v)) \cdot \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv \leftarrow \text{belopp!}$$

Rita!



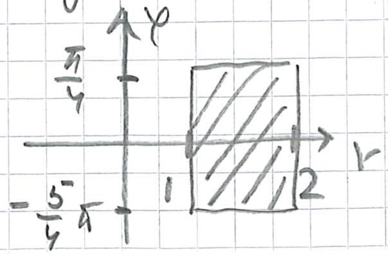
(i2 för r)

Variabelbyte

$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$ (polära koordin.)

$$\frac{\partial(xy)}{\partial(r\varphi)} = r$$

Obs $D_{r\varphi} =$



$\Rightarrow$

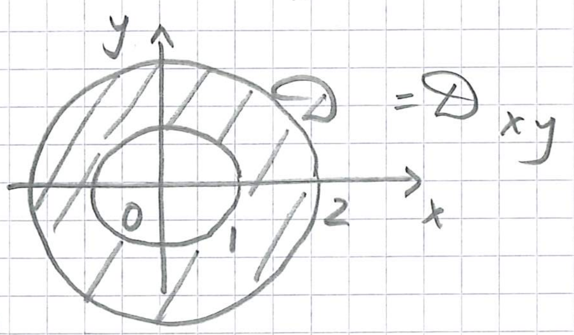
$$I = \iint_{D_{r\varphi}} \underbrace{r\cos\varphi}_{x} \cdot \underbrace{r^2}_{x^2+y^2} \cdot \underbrace{r}_{|\det|} \, dr\,d\varphi =$$

$$= \int_1^2 \left(\int_{-\frac{5}{4}\pi}^{\pi/4} r^4 \cdot \cos\varphi \, d\varphi \right) dr = \int_1^2 r^4 \sin\varphi \Big|_{-\frac{5}{4}\pi}^{\pi/4} dr =$$

$$= \int_1^2 r^4 \cdot 2 \cdot \frac{1}{\sqrt{2}}\, dr = \sqrt{2} \cdot \frac{r^5}{5}\Big|_1^2 = \frac{\sqrt{2}}{5}(32-1) =$$

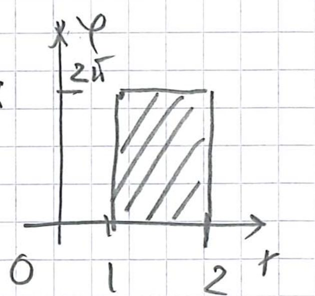
$$= \frac{\sqrt{2}}{5} \cdot 31$$

(b) $I = \iint_D x^2 \ln(x^2+y^2)\,dx\,dy$, $D = \{1 \le x^2+y^2 \le 4\}$



Variabelbyte:

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \Rightarrow D_{r\varphi}:$$



$$\Rightarrow I = \iint_{D_{r\varphi}} r^2\cos^2\varphi \cdot \ln(r^2) \cdot \underset{=\,\underline{\text{obs}}\,!}{r}\, dr\,d\varphi =$$

$$= \int_1^2 \left(\int_0^{2\pi} r^3 \ln(r^2) \cdot \cos^2\varphi \, d\varphi \right) dr =$$

$$\int_1^2 r^3 \ln(r^2) \left(\underbrace{\int_0^{2\pi} \left(\frac{1+\cos 2\varphi}{2} \right) d\varphi}_{\pi} \right) dr = \pi \int_1^2 r^3 \ln r^2 \, dr =$$

$$= \left| \begin{matrix} t = r^2 \\ dt = 2r\,dr \end{matrix} \right| = \pi \int_{r=1}^{r=2} t \ln t \cdot \frac{dt}{2} =$$

$$= \frac{\pi}{2} \int_{r=1}^{r=2} \overset{\frac{t^2}{2}}{\overset{\uparrow}{t}}\, \underset{\frac{1}{t}}{\underset{\downarrow}{\ln t}}\, dt = \left| \begin{matrix} \frac{t^2}{2} \ln t - \int \frac{t}{2}\,dt = \\ = \frac{t^2}{2} \ln t - \frac{1}{4} t^2 + c \end{matrix} \right|$$

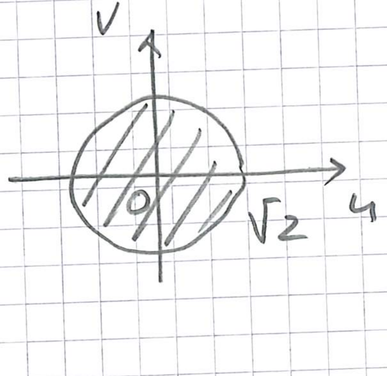
$$= \frac{\pi}{2} \left(\frac{r^4}{2} \ln r^2 - \frac{1}{4} r^4 \right) \Big|_1^2 = \frac{\pi}{2} \left(16 \ln 2 - 4 + \frac{1}{4} \right)$$

$$= 8\pi \ln 2 - \frac{15}{8} \pi$$

5.25. $I = \iint_{D} xy\,dx\,dy \qquad D = \left\{ (x-2)^2 + (y+1)^2 \leq 2 \right\}$

I:a variabelbyte $\begin{cases} u = x-2 \\ v = y+1 \end{cases}$ eller $\begin{cases} x = u+2 \\ y = v-1 \end{cases}$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \;; \quad D_{uv}:$

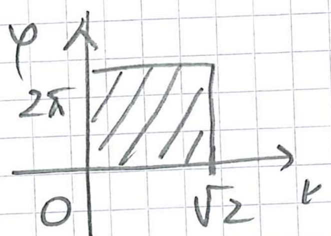


$$\Rightarrow I = \iint_{D_{uv}} (u+2)(v-1) \cdot 1\,du\,dv =$$

$$= \underbrace{\iint_{D_{uv}} uv\,du\,dv}_{I_1} + 2 \overset{=0}{\iint_{D_{uv}} v\,du\,dv} - \overset{=0}{\iint_{D_{uv}} u\,du\,dv} - 2 \cdot \overbrace{\iint_{D_{uv}} 1 \cdot du}^{=2\pi}$$

$$I_1 = \left| \begin{matrix} u = r\cos\varphi \\ v = r\sin\varphi \end{matrix} \right| = \iint_{\mathcal{D}_{r\varphi}} r\cos\varphi \, r\sin\varphi \cdot r \, dr \, d\varphi =$$

$= \Big|$



$\overset{\shortparallel}{\mathcal{D}_{r\varphi}}$

$\Big| = \int_0^{\sqrt2} \left(\int_0^{2\pi} r^3 \cos\varphi \sin\varphi \, d\varphi \right) dr$

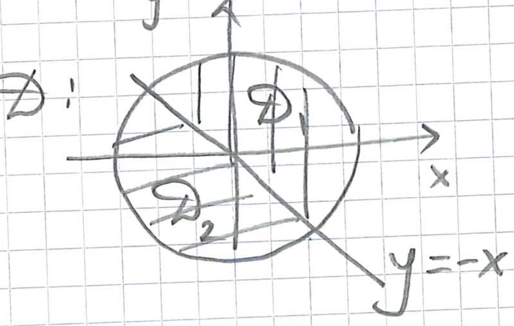
$/t = \sin\varphi/$

$$= \int_0^{\sqrt3} r^3 \cdot \frac{\sin^2\varphi}{2} \Big|_0^{2\pi} dr = 0$$

$$\Rightarrow \quad I = 0 + 0 + 0 - 2 \cdot 2\pi = -4\pi$$

$\underline{5.26}$ $\quad I = \iint_{\mathcal{D}} |x+y| \, dx dy, \quad \mathcal{D} = \{x^2 + y^2 \le 1\}.$

$\underline{\text{Obs}}$ $\quad |x+y| = \begin{cases} x+y & \text{om } x+y \ge 0 \Leftrightarrow y \ge -x \\ -(x+y) & \text{om } x+y \le 0 \Leftrightarrow y \le -x \end{cases}$



$\underline{\text{Obs}}$ $\quad \mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$

$\underline{0}$

$$I = \underset{I_1}{\underset{\shortparallel}{\iint_{\mathcal{D}_1}}} + \underset{I_2}{\underset{\shortparallel}{\iint_{\mathcal{D}_2}}}$$

$$I_1 = \left| \begin{matrix} x = r\cos\varphi \\ y = r\sin\varphi \end{matrix} \right| = \iint_{\mathcal{D}_1} (x+y) \, dx dy =$$

$$\int_0^1 \left(\int_{-\frac{\pi}{4}}^{3\pi/4} (r\cos\varphi + r\sin\varphi)\, r\, dr \right) d\varphi =$$

$$\int_0^1 r^2 \sin\varphi \Big|_{-\pi/4}^{3\pi/4} dr + \int_0^1 r^2(-\cos\varphi) \Big|_{-\pi/4}^{3\pi/4} dr =$$

$$= \sqrt{2}\cdot\frac{1}{3} + \sqrt{2}\cdot\frac{1}{3} = \frac{2\sqrt{2}}{3}$$

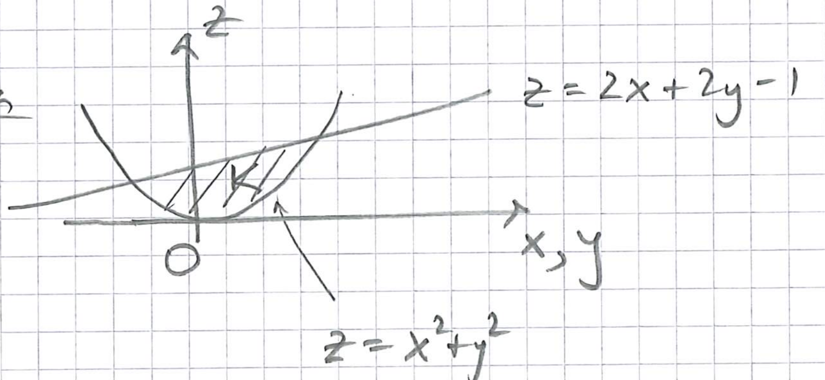
På samma vis $I_2 = \frac{2\sqrt{2}}{3}$

$\Rightarrow I = I_1 + I_2 = \frac{4\sqrt{2}}{3}$

5.29 $\quad z = 2x + 2y - 1$ (ett plan)

$z = x^2 + y^2$ (en paraboloid)

Rita



$$\text{Volym av } K = \iint_D \left((2x + 2y - 1) - (x^2 + y^2) \right) dx\,dy$$

$D = ?$ Obs D avgränsas av kurvan

$$2x + 2y - 1 = x^2 + y^2 \quad \text{eller}$$

$$x^2 - 2x + y^2 - 2y + 1 = 0 \quad \text{eller}$$

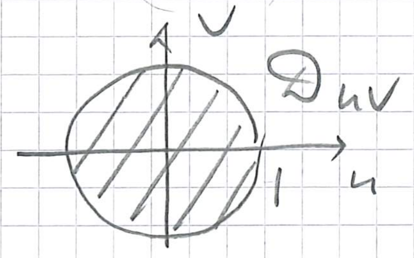
Kvadratkomplettera!

$(x-1)^2+(y-1)^2=1$ (en cirkel med radie 1 o centrum i $(1,1)$)

$\Rightarrow \mathcal{D}=\{(x-1)^2+(y-1)^2\le 1\}.$

<u>1:a variabelbyte</u> $\begin{cases}u=x-1\\ v=y-1\end{cases} \Leftrightarrow \begin{cases}x=u+1\\ y=v+1\end{cases}$

det $\dfrac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}1&0\\0&1\end{vmatrix}=1$ $\Rightarrow V=\iint_{\mathcal{D}_{uv}}(1-u^2-v^2)\,du\,dv=$



$= \left|\begin{array}{l} u=r\cos\varphi \\ v=r\sin\varphi \\ \text{2:a subst.}\end{array}\right| = \iint_{\mathcal{D}_{r\varphi}}(1-r^2)\,r\,dr\,d\varphi=$

$=\int_0^1\left(\int_0^{2\pi}(r-r^3)\,d\varphi\right)dr = 2\pi\left(\frac{r^2}{2}-\frac{r^4}{4}\right)\Big|_0^1=$

$=2\pi\cdot\frac{1}{4}=\frac{\pi}{2}$

<u>5.32</u> $I=\iint_{\mathcal{D}} y\,dx\,dy$, där

$\mathcal{D}=\{0\le \underbrace{x-2y}_{u}\le 2,\ -1\le \underbrace{x+y}_{v}\le 1\}.$

<u>substit.</u>

Obs

$$\begin{cases} u = x - 2y & (1) \\ v = x + y & (2) \end{cases} \qquad \frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{\frac{\partial(u,v)}{\partial(x,y)}} =$$

$$= \frac{1}{\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{1}{3};$$

(2) − (1):

$$\frac{v-u}{3} = y$$

Obs $\mathcal{D}_{uv} = \{ 0 \leq u \leq 2, -1 \leq v \leq 1 \} \Rightarrow$

$$I = \iint\limits_{\mathcal{D}_{uv}} \left(\frac{v-u}{3}\right) \frac{1}{3} \, du \, dv = \frac{1}{9} \iint\limits_{\mathcal{D}_{uv}} (v-u) \, du \, dv =$$

$$= \frac{1}{9} \int_0^2 \left(\int_{-1}^{1} (v-u) \, dv \right) du = \frac{1}{9} \int_0^2 \left(\frac{v^2}{2} - uv \right) \Big|_{-1}^{1} du =$$

$$= \frac{1}{9} \int_0^2 -2u \, du = -\frac{2}{9} \frac{u^2}{2} \Big|_0^2 = -\frac{4}{9}$$