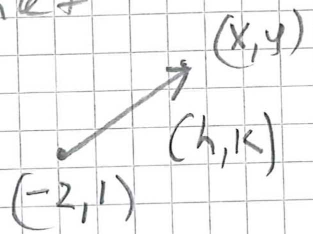
1

(1) $f(x,y) = x^2y^3 - 5xy + x^3 - 5y^2 + 1, \quad P(-2,1)$

$P_1(h,k) = f(P) + f_x'(P)\cdot h + f_y'(P)\cdot k,$ $\quad h = x+2$, $k = y-1$

$P_2(h,k) = P_1(h,k) + \frac{1}{2}\left(f_{xx}''(P)h^2 + \underline{\underline{2}}\cdot f_{xy}''(P)hk + f_{yy}''(P)k^2\right)$

<u>Obs</u>



$f(P) = (-2)^2\cdot 1^3 - 5(-2)\cdot 1 + (-2)^3 - 5\cdot 1^2 + 1 =$

$= \underbrace{4 + 10}_{14} - \underbrace{8 - 5}_{-13} + 1 = \underline{\underline{2}}$

$f_x' = 2xy^3 - 5y + 3x^2$

$f_y' = 3x^2y^2 - 5x - 10y$

$f_x'(P) = 2\cdot(-2)\cdot 1 - 5 + 3\cdot(-2)^2$
$= -4 - 5 + 12 = \underline{\underline{3}}$

$f_y'(P) = 3(-2)^2 - 5\cdot(-2) - 10 = 3\cdot 4 + 10 - 10 = \underline{\underline{12}}$

$\Rightarrow P_1(h,k) = \underline{2 + 3h + 12k}$

$f_{xx}'' = 2y^3 + 6x$

$f_{xy}'' = 3\cdot 2\cdot xy^2 - 5 = 6xy^2 - 5$

$f_{yy}'' = 6x^2y - 10$

$f_{xx}''(P) = 2 + 6\cdot(-2) = -10$

$f_{xy}''(P) = 6\cdot(-2) - 5 = -17$

$f_{yy}''(P) = 6\cdot 4 - 10 = 14$

$P_2(h,k) = P_1(h,k) + \frac{1}{2}\left(-10h^2 - 34hk + 14k^2\right) =$

$$P_2(h,k) = 2 + 3h + 12k - 5h^2 - 17hk + 7k^2$$

(2) $\frac{\partial z}{\partial x} - \frac{1}{2}\frac{\partial z}{\partial y} = \cancel{x + 2y + 11}$ $5x - y + 11$ lös ekv.

(i) $\begin{cases} u = x + 2y \\ v = 4x - 3y \end{cases}$

$u'_x = 1,\ u'_y = 2$

$v'_x = 4,\ v'_y = -3$

$z'_x = z'_u \cdot \underbrace{u'_x}_{=1} + z'_v \cdot \underbrace{v'_x}_{=4} = z'_u + 4z'_v$

$z'_y = z'_u \cdot \underbrace{u'_y}_{=2} + z'_v \cdot \underbrace{v'_y}_{=-3} = 2z'_u - 3z'_v$

$\Rightarrow (z'_u + 4z'_v) - \frac{1}{2}(2z'_u - 3z'_v) = \cancel{u + 11}$ $u + v + 11$

eller $\frac{11}{2} z'_v = \cancel{u + 11}$ $u + v + 11$ eller $z'_v = \cancel{\frac{2}{11}u + 2}$ $\frac{2}{11}(u+v) + 2$

(ii) $\cancel{z = \int z'_v\, dv = \left(\frac{2}{11}u + 2\right)v + c(u)}$

$\Rightarrow \cancel{z(x,y) = \left(\frac{2}{11}(x+2y) + 2\right)(4x - 3y) + c(x+2y)}$

Istället! $\boxed{\frac{\partial z}{\partial x} - \frac{1}{2}\frac{\partial z}{\partial y} = 5x - y + 11}$

$\Rightarrow \frac{11}{2} z'_v = (u+v) + 11$ eller

$z'_v = \frac{2}{11}(u+v) + 2$ $\Rightarrow$ Integrera m.a.v. V!

a.

$$z = \int z'_v \, dv = \frac{2}{11}\int (u+v)\,dv + \int 2\,dv =$$

$$= \frac{2}{11}\left(\left(uv + \frac{v^2}{2}\right)\right) + 2v + C(u)$$

Bryta ut $\frac{2}{11} \cdot v$

$$\Rightarrow z(x,y) = \frac{2}{11}\left((x+2y) + \frac{4x-3y}{2} + 11\right)(4x-3y) + C(x+2y)$$

$$= \frac{2}{11}\left(3x + \frac{1}{2}y + 11\right)(4x-3y) + C(x+2y)$$

(3) $f(x,y) = 2y^3 + 5y^2 + yx^2 + x^2 + 3$

(i) Stat. punkter:

$$\begin{cases} f'_x = 0 \\ f'_y = 0 \end{cases} \qquad \begin{cases} 2xy + 2x = 0 & (1) \\ 6y^2 + 10y + x^2 = 0 & (2) \end{cases}$$

(1): $2x(y+1) = 0$

(a) $x = 0 \xrightarrow{} (2)$ $6y^2 + 10y = 0$ eller $3y^2 + 5y = 0$

$y_1 = 0$, $y_2 = -5/3$

$\Rightarrow P_1(0,0)$, $P_2(0,-5/3)$

(b) $y = -1 \xrightarrow{} (2)$ $6 - 10 + x^2 = 0 \Leftrightarrow x^2 = 4$ $x_{1,2} = \pm 2$

$\Rightarrow P_3(2,-1)$, $P_4(-2,-1)$

Obs

P: $Q(h,k) = f''_{xx}(P)h^2 + \underline{\underline{2}} f''_{xy}(P)hk + f''_{yy}(P)k^2$

		P_1	P_2	P_3	P_4
f''_{xx}	$= 2y+2$	2	$-\frac{10}{3}+2 = -\frac{4}{3}$	0	0
f''_{xy}	$= 2x$	0	0	4	-4
f''_{yy}	$= 12y+10$	10	$\frac{\cancel{12}^{4}\cdot(-5)}{\cancel{3}}+10 = -10$	-2	-2

$Q_1(h,k) = 2h^2 + 10k^2$, pos. def, $\Rightarrow P_1$ är en st. lok. minimum

$Q_2(h,k) = -\frac{4}{3}h^2 - 10k^2$, neg. def $\Rightarrow P_2$ är en st. lok maximum

$Q_3(h,k) = 2\cdot 4hk - 2k^2 = -2(k^2 - 4hk) =$
$= -2\left((k-2h)^2 - 4h^2\right) = -2(k-2h)^2 + 8h^2$

Obs $Q_3(1,2) = 8 > 0$, $Q_3(0,1) = -2 < 0 \Rightarrow$ indef. $\Rightarrow P_3$ är en sadelpunkt

$Q_4(h,k) = -8hk - 2k^2 = -2(k^2 + 4hk) =$
$= -2\left((k+2h)^2 - 4h^2\right) = -2(k+2h)^2 + 8h^2$

$\left.\begin{array}{l} Q_4(1,-2) = 8 > 0 \\ Q_4(0,1) = -2 < 0 \end{array}\right\} \Longrightarrow$ indef $\Rightarrow P_4$ är en sadelpunkt.

(5)

(4) $f(x,y,z) = x+y+z$ då $\begin{cases} x^2+y^2+z^2=2 \\ x^2+y^2=z \end{cases}$

optimera!

Obs en cirkel (?)

max & min finns!

Inför: $g = x^2+y^2+z^2$

$\underline{h = x^2+y^2-z}$

Obs

$\begin{cases} \nabla f, \nabla g, \nabla h \text{ är lin. beroende} & (1) \\ g=2 & (2) \\ h=0 & (3) \end{cases} \iff$ $2-z^2=z$, $z=\cancel{-2},1$ $\Rightarrow \begin{cases} x^2+y^2=1 \\ z=1 \end{cases}$

(1): $\begin{vmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 2x & 2y & -1 \end{vmatrix} = 0$ eller $\begin{vmatrix} 1 & 1 & 1 \\ 2x & 2y & 2z \\ 0 & 0 & -1-2z \end{vmatrix} = 0$

eller $-(1+2z)\cdot(2y-2x)=0$ eller

$2(x-y)(1+2z)=0$

(a) $x=y$ (b) $z=-1/2$

(a): $\begin{cases} 2x^2+z^2=2 \ (2) \\ 2x^2=z \geq 0 \ (3) \end{cases}$ eller $2-z^2=z$ eller $z^2+z-2=0$

$z_{1,2} = -2, 1$ Obs $\underline{\underline{z=1}}$

Obs

$\Rightarrow 2x^2=1 \quad x=\pm\frac{1}{\sqrt{2}} \Rightarrow \underline{P_1\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},1\right), P_2\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},1\right)}$

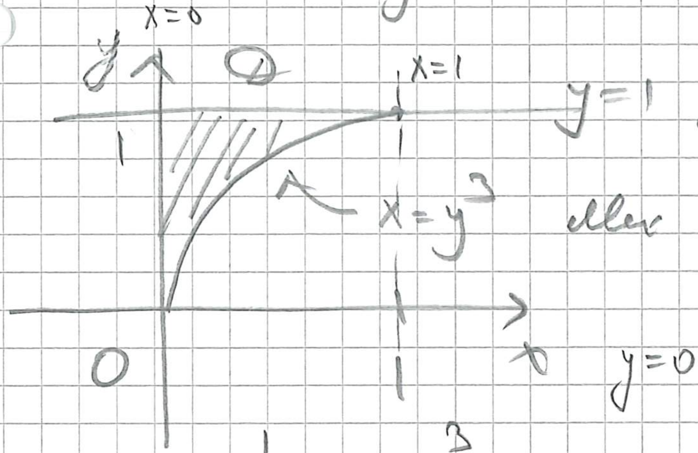
(b) $z=-\frac{1}{2}$ <u>passar ej</u> $x^2+y^2=-\frac{1}{2}$ (ekv (3)) ⑥

saknar lösning

$f(P_1)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+1=1+\sqrt{2}$ <u>max</u>

$f(P_2)=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+1=-\sqrt{2}+1$ <u>min</u>

⑤ $$I=\int_{x=0}^{x=1}\left(\int_{y=\sqrt[3]{x}}^{y=1} e^{x/y^2}\,dy\right)dx=\iint_D e^{x/y^2}\,dx\,dy$$



$\Leftarrow$ <u>Rita!</u>

$$I=\int_0^1\left(\int_0^{y^3} e^{x/y^2}\,dx\right)dy=\left|\begin{array}{l} t=\frac{x}{y^2} \\ dt=\frac{1}{y^2}dx \end{array}\right| dx=y^2\,dt$$

$$=\int_0^1 y^2\left(\int_{x=0}^{x=y^3} e^t\,dt\right)dy=\int_0^1 y^2 e^t\Big|_{0=x}^{x=y^3}dy=\left|\begin{array}{l}\text{tillbaka}\\ \text{till } x\end{array}\right|$$

$$=\int_0^1 y^2 e^{x/y^2}\Big|_0^{y^3}dy=\int_0^1 y^2(e^y-1)\,dy=$$

$$\underbrace{\int_0^1 y^2e^y\,dy}_{I_1}-\underbrace{\int_0^1 y^2\,dy}_{I_2}=?$$

7

$$I_1 = \int_0^1 \underset{\downarrow\, 2y}{y^2}\, \overset{e^y\,\uparrow}{e^y}\, dy = \left|\; p.\ i.: \int \underset{\downarrow\, f}{F}\, \overset{G\,\uparrow}{g} = FG - \int f G \right.$$

$$= y^2 e^y - \int 2y\, e^y\, dy = y^2 e^y - 2\left(y e^y - \int e^y\, dy\right) =$$

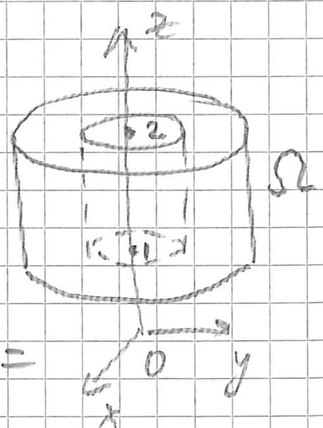
$$= y^2 e^y - 2y e^y + 2e^y + c$$

$$\Rightarrow I_1 = \left(y^2 e^y - 2y e^y + 2e^y\right)\Big|_0^1 =$$

$$= (e^1 - \underbrace{2e^1 + 2e^1}_{=0}) - (0 - 0 + 2\cdot 1) = e - 2$$

$$I_2 = \int_0^1 y^2\, dy = \frac{y^3}{3}\Big|_0^1 = \frac{1}{3} \Rightarrow I = e - 2 - \frac{1}{3} = e - 7/3$$

6 $\quad M = \iiint_\Omega f(x, y, z)\, dx\, dy\, dz =$



$$= \iint_D \left(\int_{1=z}^{2=z} z^3 \cdot \ln(x^2+y^2)\, dz \right) dx\, dy =$$

$D: (4 \le x^2 + y^2 \le 9)$

$$= \iint_D \ln(x^2+y^2) \cdot \frac{z^4}{4}\Big|_1^2\, dx\, dy = \frac{1}{4} \iint_D \ln(x^2+y^2) \cdot (\overbrace{2^4 - 1}^{15})\, dx\, dy$$

$$= \frac{15}{4} \int_2^3 \left(\int_0^{2\pi} \underbrace{2r \cdot \ln r}_{\text{Obs}}\, d\varphi \right) dr = \frac{15}{4} \cdot 4\pi \int_2^3 r \ln r\, dr =$$

Obs $\quad \frac{\partial xy}{\partial r\varphi} = r$

$$= 15\pi \int_2^3 r \ln r \, dr = ?$$

$$\int \overset{r^2/2}{\underset{}{r}} \underset{\frac{1}{r}}{\ln r} \, dr \overset{p.i}{=} \frac{r^2}{2} \ln r - \int \frac{r^2}{2} \cdot \frac{1}{r} \, dr =$$

$$= \frac{r^2}{2} \ln r - \int \frac{r}{2} \, dr = \frac{r^2}{2} \ln r - \frac{r^2}{4} + c$$

$$= 15\pi \left(\frac{r^2}{2} \ln r - \frac{r^2}{4} \right) \Bigg|_2^3 = 15\pi \left(\left(\frac{9}{2} \ln 3 - \frac{9}{4} \right) - \right.$$

$$\left. - \left(\frac{4}{2} \ln 2 - \frac{4}{4} \right) \right) = 15\pi \left(\frac{9}{2} \ln 3 - 2 \ln 2 - \frac{9}{4} + 1 \right)$$

$$= \underline{15\pi \left(\frac{9}{2} \ln 3 - 2 \ln 2 - \frac{5}{4} \right)}$$