2.4 Bestäm differentialen till

$f(x,y)$: $\underline{df = f'_x \cdot dx + f'_y \cdot dy}$ (Kort)

Eller differentialen till $f(x,y)$ i punkten (a,b):

$$\underline{df(a,b)(h,k) = f'_x(a,b)\cdot h + f'_y(a,b)\cdot k}$$

(a) $f(x,y) = \sin(xy^2)$, $f'_x = \cos(xy^2)\cdot(xy^2)'_x =$

$= y^2\cos(xy^2)$, $f'_y = \cos(xy^2)\cdot(xy^2)'_y =$

$= 2xy\cos(xy^2)$

$\Rightarrow df = y^2\cos(xy^2)dx + 2xy\cos(xy^2)dy =$

$= y\cos(xy^2)(y\,dx + 2x\,dy)$

Eller $df(a,b)(h,k) = b\cos(ab^2)(b\cdot h + 2ak)$

(c) $f(x,y,z) = \sin(xyz)$,

$f'_x = yz\cos(xyz)$, $f'_y = xz\cos(xyz)$, $f'_z = xy\cos(xyz)$

$\Rightarrow df = \cos(xyz)(yz\,dx + xz\,dy + xy\,dz)$

(d) $f(p,V,T) = \dfrac{pV}{T}$,

$f'_p = \dfrac{V}{T}$, $f'_V = \dfrac{p}{T}$, $f'_T = -\dfrac{pV}{T^2}$ $\Rightarrow$

$df = \dfrac{1}{T}\left(V\,dp + p\,dV - \dfrac{pV}{T}dT\right)$

2.5 Bestäm felgränsen G_F:

$P(x,y,z)$, $P_0(x_0,y_0,z_0)$, $f(x,y,z)$

$$F\,(\text{fel}) = f(P) - f(P_0) \approx f'_x(P_0)(x-x_0) + $$
$$+ f'_y(P_0)(y-y_0) + f'_z(P_0)(z-z_0) \qquad (*)$$

Data: $x = 2.00$, $y = 3.00$, $z = 4.00$ är korrekt avrundade (till 2 decimaler)

Det tolkar man så här:

För P: $1.995 \le x \le 2.005$, $2.995 \le y \le 3.005$
$3.995 \le z \le 4.005$

För P_0: $x_0 = 2$, $y_0 = 3$, $z_0 = 4$

Inför $d = 0.005$ o Obs $|x-x_0| \le d$, $|y-y_0| \le d$ (belopp)

$|z-z_0| \le d$.

För uppskattning av G_F: (använd $(*)$)

$$|F| \lesssim |f'_x(P_0)| \cdot |x-x_0| + |f'_y(P_0)| \cdot |y-y_0| +$$
$$+ |f'_z(P_0)| \cdot |z-z_0| \lesssim \underbrace{\left(|f'_x(P_0)| + |f'_y(P_0)| + |f'_z(P_0)|\right) \cdot d}_{G_F}$$

(a) $f(x,y,z) = 3x + y - z$,

$f'_x = 3$, $f'_y = 1$, $f'_z = -1$

$\Rightarrow G_F = (3 + 1 + |-1|) \cdot 0.005 = 5 \cdot 0.005 = 0.025$

(b) $f(x,y,z) = \frac{x \cdot y}{z}$

$f'_x = \frac{y}{z}$, $f'_y = \frac{x}{z}$, $f'_z = -\frac{xy}{z^2}$

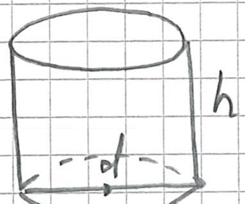
$f'_x(P_0) = \frac{3}{4}$, $f'_y(P_0) = \frac{2}{4} = \frac{1}{2}$, $f'_z = -\frac{2 \cdot 3}{4^2} = -\frac{3}{8}$

$G_F = \left(\frac{3}{4} + \frac{1}{2} + \left|-\frac{3}{8}\right|\right) \cdot 0.005 = (0.75 + 0.5 + 0.375) \cdot$

$\cdot 0.005 = 1.625 \cdot \frac{1}{2} \cdot 10^{-2}$

(c) Analogt som ovan.

2.6



V (volym) $= \frac{\pi \cdot d^2}{4} \cdot h = V(d,h)$

Inför $\Delta V = V(d,h) - V(d_0,h_0)$ (fel)

Relativt fel: $\frac{\Delta V}{V(d_0,h_0)} \cdot 100\%$

Inför: $d - d_0 = \Delta d$, $h - h_0 = \Delta h$

Data: $d = 16 \pm 0.4$, $h = 8 \pm 0.1$

Det tolkas så här: $\begin{cases} |\Delta d| \leq 0.4, & |\Delta h| \leq 0.1 \\ d_0 = 16, & h_0 = 8 \end{cases}$

Använd (*) från 2.5:

$V'_d = \frac{1}{2}\pi d h$, $V'_h = \frac{\pi d^2}{4}$, $V'_d(d_0,h_0) = \frac{1}{2}\pi \cdot 16 \cdot 8 =$

$= \pi \cdot 64$, $V'_h(d_0,h_0) = \frac{\pi \cdot 16^2}{4} = \pi \cdot 64$

$$|\Delta V| \lesssim |V_d'(d_0,h_0)|\cdot|\Delta d| + |V_h'(d_0,h_0)|\cdot|\Delta h|$$

$$\lesssim \pi\cdot 64\cdot 0.4 + \pi\cdot 64\cdot 0.1 = 32\pi, \quad V(d_0,h_0) = \frac{\pi\cdot 16^2}{4}\cdot 8 =$$

Den relativa felgränsen: $= 512\pi$

$$\frac{32\pi}{512\pi}\cdot 100\% = \frac{1}{16}\cdot 100\%$$

<u>2.14</u> Betrakta $h(x,y,z) = f\left(\frac{x}{y}, \frac{y}{z}\right)$, där

f är en två variabel funktion $f(u,v)$.

Beräkna $x\cdot\frac{\partial h}{\partial x} + y\frac{\partial h}{\partial y} + z\cdot\frac{\partial h}{\partial z} = U$

$$\frac{\partial h}{\partial x} = /\text{kedjeregeln}/ = f_u'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{x}{y}\right)_x' +$$

$$+ f_v'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{y}{z}\right)_x' = \frac{1}{y}\cdot f_u'\left(\frac{x}{y},\frac{y}{z}\right)$$

$$\frac{\partial h}{\partial y} = f_u'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{x}{y}\right)_y' + f_v'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{y}{z}\right)_y' =$$

$$= f_u'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(-\frac{x}{y^2}\right) + \frac{1}{z}\cdot f_v'\left(\frac{x}{y},\frac{y}{z}\right)$$

$$\frac{\partial h}{\partial z} = f_u'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{x}{y}\right)_z' + f_v'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(\frac{y}{z}\right)_z' = f_v'\left(\frac{x}{y},\frac{y}{z}\right)\cdot\left(-\frac{y}{z^2}\right)$$

$$\Rightarrow U = x\cdot\frac{1}{y}\cdot f_u' + y\cdot f_u'\cdot\left(-\frac{x}{y^2}\right) + y\cdot\frac{1}{z}\cdot f_v' + z\cdot f_v'\cdot\left(-\frac{y}{z^2}\right)$$

$$= \frac{x}{y}\cdot f_u' - \frac{x}{y}\cdot f_u' + \frac{y}{z}f_v' - \frac{y}{z}\cdot f_v' = 0$$