**Examiner**: Xiangfeng Yang (Tel: 013-285788). **Things allowed (Hjälpmedel)**: a calculator.
**Scores rating (Betygsgränser)**: 8-11 points giving rate 3; 11.5-14.5 points giving rate 4; 15-18 points giving rate 5.

# 1 (3 points)

Suppose that a box contains 10 balls of which 6 balls are white and 4 balls are black. Now 3 balls are randomly taken out from the box in two different ways:
(1.1) (1p) without replacement (i.e. once a ball is chosen then it will be put outside of the box), if $X$ denotes the number of white balls in these 3 chosen balls, then what is $P(X=2)$?
(1.2) (2p) with replacement (i.e. once a ball is chosen then it will be put back into the box), if $Y$ denotes the number of white balls in these 3 chosen balls, then what is $P(Y=2)$?

*Solution.* (1.1) $X \sim$ Hypergeometric, and it follows that

$$P(X=2) = \frac{\binom{6}{2}\binom{4}{1}}{\binom{10}{3}} = \frac{15 \cdot 4}{120} = 0.5.$$

(1.2) $Y \sim Bin(3, 0.6)$, and it follows that

$$P(Y=2) = \binom{3}{2} 0.6^2 (1-0.6)^1 = 0.432.$$

□

# 2 (3 points)

A fair die (with six sides marked as $1, 2, 3, 4, 5$ and 6) is tossed, and let $X$ be the upper side number.
(2.1) (1p) Find the mean $\mu = E(X)$ and variance $\sigma^2 = V(X)$ of $X$.
(2.2) (2p) If the die is tossed 100 times, and let $X_1, X_2, \ldots, X_{100}$ denote these 100 upper side numbers. Find the probability $P(X_1 + X_2 + \ldots + X_{100} \leq 400)$.

*Solution.* (2.1) The mean is

$$\mu = E(X) = 1 \cdot \frac{1}{6} + \ldots + 6 \cdot \frac{1}{6} = 3.5.$$

The variance is

$$\sigma^2 = V(X) = E(X^2) - \mu^2 = 1^2 \cdot \frac{1}{6} + \ldots + 6^2 \cdot \frac{1}{6} - 3.5^2 = 15.16667 - 12.25 = 2.91667.$$

(2.2)

$$\begin{aligned} &P(X_1 + X_2 + \ldots + X_{100} \leq 400) = P(\bar{X} \leq 4) \\ &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \leq \frac{4-\mu}{\sigma/\sqrt{n}}\right) = P\left(N(0,1) \leq \frac{4-3.5}{\sqrt{2.91667}/\sqrt{100}}\right) \\ &= P(N(0,1) \leq 2.93) = \Phi(2.93) = 0.9983. \end{aligned}$$

□

## 3 (3 points)

Let $(X, Y)$ be a two dimensional continuous random variable with a joint probability density function

$$f(x,y) = \begin{cases} c \cdot e^{-(x+y)}, & \text{if } 0 < x < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

(3.1) (1p) Find the value of $c$ so that $f(x, y)$ is indeed a joint probability density function.
(3.2) (1p) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$ for $X$ and $Y$.
(3.3) (1p) Find the probability $P(X + 1 \leq Y)$.

*Solution.* (3.1)

$$\begin{aligned} 1 &= \int\int_{\{0<x<y<\infty\}} f(x,y)dxdy = \int_0^\infty \left(\int_0^y c \cdot e^{-x}e^{-y}dx\right)dy = c \cdot \int_0^\infty e^{-y}\left(\int_0^y e^{-x}dx\right)dy \\ &= c \cdot \int_0^\infty e^{-y}\left(1 - e^{-y}\right)dy = c \cdot \int_0^\infty e^{-y}dy - c \cdot \int_0^\infty e^{-2y}dy = c - c/2 = c/2 \Rightarrow c = 2. \end{aligned}$$

(3.2)

$$\begin{aligned} f_X(x) &= \int_x^\infty f(x,y)dy = ce^{-x}\int_x^\infty e^{-y}dy = ce^{-2x}, \text{ for } x > 0, \\ f_Y(y) &= \int_0^y f(x,y)dx = ce^{-y}\int_0^y e^{-x}dx = ce^{-y}(1 - e^{-y}), \text{ for } y > 0. \end{aligned}$$

(3.3)

$$\begin{aligned} P(X+1 \leq Y) &= \int_1^\infty \left(\int_0^{y-1} c \cdot e^{-x}e^{-y}dx\right)dy = c \cdot \int_1^\infty e^{-y}\left(\int_0^{y-1} e^{-x}dx\right)dy \\ &= c \cdot \int_1^\infty e^{-y}\left(1 - e^{-(y-1)}\right)dy = c \cdot \int_1^\infty e^{-y}dy - c \cdot e \cdot \int_1^\infty e^{-2y}dy \\ &= c/e - c/(2e) = c/(2e) = 1/e. \end{aligned}$$

□

## 4 (3 points)

A population $X$ is a continuous random variable with a probability density function as follows:

$$f(x) = \begin{cases} \frac{1}{2}(1+\theta x), & \text{if } -1 < x < 1 \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in [-1, 1]$ is an unknown parameter. A sample $\{0.2, -0.6\}$ is taken from this population.
(4.1) (1p) Estimate $\theta$ by $\hat{\theta}_{MM}$ using the Method of Moments.
(4.2) (2p) Estimate $\theta$ by $\hat{\theta}_{ML}$ using the Maximum-Likelihood method.

*Solution.* (4.1) For MM, we use the first equation $E(X) = \bar{x}$, where

$$E(X) = \int_{-1}^1 x \cdot f(x)dx = \frac{1}{2}\int_{-1}^1 xdx + \frac{1}{2} \cdot \theta \int_{-1}^1 x^2dx = \frac{\theta}{3}.$$

Theretofore, $\frac{\theta}{3} = \bar{x}$ which implies that $\hat{\theta}_{MM} = 3\bar{x} = 3 \cdot (-0.2) = -0.6$.
(4.2) For ML, the likelihood function is

$$L(\theta) = f(x_1) \cdot f(x_2) = \frac{1}{2}(1+\theta x_1) \cdot \frac{1}{2}(1+\theta x_2) = \frac{1}{4}(1+\theta x_1)(1+\theta x_2).$$

The log likelihood function is

$$\ln L(\theta) = \ln\frac{1}{4} + \ln(1+\theta x_1) + \ln(1+\theta x_2).$$

Then

$$\ln' L(\theta) = \frac{x_1}{1+\theta x_1} + \frac{x_2}{1+\theta x_2} = \frac{0.2}{1+0.2\theta} - \frac{0.6}{1-0.6\theta} = \frac{-0.4-0.24\theta}{(1+0.2\theta)(1-0.6\theta)} < 0, \text{ for } \theta \in [-1,1].$$

Therefore $\ln L(\theta)$ is decreasing for $\theta \in [-1, 1]$, so the maximum of $\ln L(\theta)$ is reached when $\theta = -1$, that is $\hat{\theta}_{ML} = -1$. □

## 5 (3 points)

For a population $X \sim N(\mu, \sigma^2)$ with $\sigma^2 = 3^2$, the $(1-\alpha)$ confidence interval of $\mu$ is given by

$$I_\mu = \left(\bar{X} - z_{\alpha/2} \cdot \sigma/\sqrt{n},\ \ \bar{X} + z_{\alpha/2} \cdot \sigma/\sqrt{n}\right) = \bar{X} \mp z_{\alpha/2} \cdot \sigma/\sqrt{n},$$

(5.1) (1p) How large should $n$ be in order that the width of 99% confidence interval of $\mu$ is at most 1?
(5.2) (2p) For $n = 16$, how large should $\alpha$ be in order that the width of $(1-\alpha)$ confidence interval of $\mu$ is at most 1? (Hint: the width of a confidence interval $I_\mu = (a,\ b)$ is equal to $b - a$).

*Solution.* (5.1) The width of the confidence interval is $2 \cdot z_{\alpha/2} \cdot \sigma/\sqrt{n}$. Therefore

$$2 \cdot z_{0.01/2} \cdot 3/\sqrt{n} \leq 1 \Leftrightarrow 2 \cdot 2.57 \cdot 3/\sqrt{n} \leq 1 \Leftrightarrow \sqrt{n} \geq 15.42 \Leftrightarrow n \geq 237.8, \text{ namely } n \geq 238.$$

(5.2) Again, the width is $2 \cdot z_{\alpha/2} \cdot \sigma/\sqrt{n}$, so

$$2 \cdot z_{\alpha/2} \cdot 3/\sqrt{16} \leq 1 \Leftrightarrow z_{\alpha/2} \leq 0.67.$$

It is from the standard normal table that the cdf corresponding to the percentile 0.67 is 0.7486. This cdf should be $(1-\alpha/2)$ (as what we have been using), namely $(1-\alpha/2) \leq 0.7486 \Leftrightarrow \alpha \geq 0.5028$.

□

## 6 (3 points)

It is claimed in a newspaper that 30% of Swedish young people prefer iPhone than Android, but we suspect there are more. A survey gave that 40 out of 100 respondents preferred iPhone. Does the survey provide any evidence that more than 30% of Swedish young people prefer iPhone than Android? We will answer this using appropriate hypotheses test with a significance level 5%.
(6.1) (1p) What are the hypotheses? (namely what are $H_0$ and $H_a$?)
(6.2) (1p) Is $H_0$ rejected based on $TS$ and $C$? Why?
(6.3) (1p) Is $H_0$ rejected based on $p$-value? Why?

*Solution.* (6.1) Let $p$ be the true proportion of Swedish young people who prefer iPhone than Android. Then
$H_0 : p = 30\%$ against $H_a : p > 30\%$.
(6.2)

$$TS = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{40/100 - 30\%}{\sqrt{30\%(1-30\%)/100}} = 2.18, \quad C = (z_\alpha, +\infty) = (1.645, +\infty).$$

It is from $TS \in C$ that $H_0$ is rejected (namely, the survey provides evidence that more than 30% of Swedish young people prefer iPhone than Android).
(6.3) The $p$-value is:

$$p\text{-value} = P(N(0,1) > TS) = P(N(0,1) > 2.18) = 1 - \Phi(2.18) = 1 - 0.9854 = 0.0146.$$

From $p$-value $< \alpha$, it follows that $H_0$ is rejected.

□

# TAMS42: formulas and tables — by Xiangfeng Yang

## 1. Basic probability

(1.1) Conditional probability $P(A|B) = \frac{P(A\cap B)}{P(B)}$.
(1.2) Total probability $P(B) = \sum_{i=1}^{k} P(B|A_i)P(A_i)$ where $\{A_i\}$ are disjoint and $\cup_{i=1}^{k} A_i = S$.
(1.3) Bayes' Theorem $P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i)P(A_i)}$ where $\{A_i\}$ are in (1.2).

## 2. Random variables (r.v.s)

(2.1) Discrete r.v. $X$ has a pmf $p(x) = P(X = x)$ satisfying $p(x) \geq 0$ and $\sum p(x_i) = 1$,

| $X$ | $x_1$ | $x_2$ | $\cdots$ | $x_n$ | $\cdots$ |
|---|---|---|---|---|---|
| $p(x)$ | $p(x_1)$ | $p(x_2)$ | $\cdots$ | $p(x_n)$ | $\cdots$ |

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \sum x_i p(x_i)$;
Variance $\sigma_X^2 = V(X) = E(X-\mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - (\sum x_i p(x_i))^2$.
(2.2) Continuous r.v. $X$ has a pdf $f(x)$ satisfying $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$,

$$P(a < X < b) = \int_a^b f(x)dx.$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \int_{-\infty}^{\infty} xf(x)dx$;
Variance $\sigma_X^2 = V(X) = E(X-\mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \left(\int_{-\infty}^{\infty} xf(x)dx\right)^2$.
(2.3) Cumulative distribution function (cdf) of a r.v. $X$ is $F(x) = P(X \leq x)$.
(2.4) $X$ and $Y$ are r.v.s, $a, b$ and $c$ are scalars, then

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$
$$V(aX + bY + c) = a^2V(X) + b^2V(Y) + 2ab\ cov(X,Y),$$
$$E(g(X,Y)) = \begin{cases} \sum_{i,j} g(x_i,y_j)\cdot p(x_i,y_j), & \text{for discrete } (X,Y), \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\cdot f(x,y)dxdy, & \text{for continuous } (X,Y). \end{cases}$$

(2.5) • Discrete r.v. $(X,Y)$ has a joint pmf $p(x,y)$ satisfying $p(x,y) \geq 0$ and $\sum_{x_i}\sum_{y_i} p(x_i,y_i) = 1$.
The *marginal pmf* of $X$ is $p_X(x) = \sum_y p(x,y)$;
The *marginal pmf* of $Y$ is $p_Y(y) = \sum_x p(x,y)$;
$X$ and $Y$ are *independent* if $p(x,y) = p_X(x)\cdot p_Y(y)$.
• Continuous r.v. $(X,Y)$ has a joint pdf $p(x,y)$ satisfying $f(x,y) \geq 0$ and $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$.
The *marginal pdf* of $X$ is $f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$;
The *marginal pdf* of $Y$ is $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)dx$;
$X$ and $Y$ are *independent* if $f(x,y) = f_X(x)\cdot f_Y(y)$.

## 3. Several special r.v.s

(3.1) $X \sim Bin(n,p)$ has a pmf $p(x) = P(X = x) = \binom{n}{x}\cdot p^x\cdot(1-p)^{n-x}, \quad x = 0,1,2,\ldots,n.$
$E(X) = n\cdot p, \quad V(X) = n\cdot p\cdot(1-p)$.
(3.2) $X \sim Po(\lambda)$ has a pmf $p(x) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0,1,2,\ldots.$
$E(X) = \lambda, \quad V(X) = \lambda$.
(3.3) $X \sim Hypergeometric$ has a pmf $p(x) = P(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$.
(3.4) $X \sim Exp(\lambda)$ has a pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$E(X) = \frac{1}{\lambda}, \quad V(X) = \left(\frac{1}{\lambda}\right)^2$.
(3.5) $X \sim N(\mu,\sigma^2)$ has a pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty.$$

$E(X) = \mu, \quad V(X) = \sigma^2$.
(3.6) $X \sim U(a,b)$ has a pdf

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}$.

## 4. Central Limit Theorem (CLT)

Suppose that a population has mean= $\mu$ and variance= $\sigma^2$. A random sample $\{X_1, X_2, \ldots, X_n\}$ from this population is given. Then for large $n \geq 30$,

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1). \qquad (1)$$

• If the population is normal, then (1) holds for any $n$.
• Note that $\mu = E(\bar{X})$ and $(\sigma/\sqrt{n})^2 = V(\bar{X})$.

## 5. Several notations in statistics

(5.1) Sample mean: $\bar{X} = \frac{X_1+X_2+\ldots X_n}{n} = \frac{\sum X_i}{n}$; $\bar{x} = \frac{x_1+x_2+\ldots x_n}{n} = \frac{\sum x_i}{n}$.
(5.2) Sample variance:

$$S^2 = \frac{\sum(X_i-\bar{X})^2}{n-1} = \frac{1}{n-1}\left(\sum X_i^2 - \frac{(\sum X_i)^2}{n}\right); \quad s^2 = \frac{\sum(x_i-\bar{x})^2}{n-1} = \frac{1}{n-1}\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right).$$

• Capital letters $\bar{X}$ and $S^2$ refer to the objects based on random sample (therefore they are in general r.v.s), while small letters $\bar{x}$ and $s^2$ are the objects based on observations (so they are scalars).
(5.3) A point estimator of $\theta$ obtained by Methods of Moments is denoted as $\hat{\theta}_{MM}$.
(5.4) A point estimator of $\theta$ obtained by Maximum Likelihood method is denoted as $\hat{\theta}_{ML}$.

## 6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population propotion(s)).

## CI-1: $(1-\alpha)$ CI of a population mean $\mu$

case 1.1 (any $n$) If population $X \sim N(\mu, \sigma^2)$ and $\sigma^2$ is known, then $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ and

$$I_\mu = \left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

case 1.2 ($n \geq 30$) For any population $X$, it holds that $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$ and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

case 1.3 (any $n$) If population $X \sim N(\mu, \sigma^2)$ and $\sigma^2$ is unknown, then $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim T(n-1)$ and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

## CI-1′: $(1-\alpha)$ CI of the difference of two population means $\mu_X - \mu_Y$

case 1.1′ (any $n_1, n_2$) If independent populations $X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2)$, and $\sigma_X^2, \sigma_Y^2$ are known, then $\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1}+\frac{\sigma_Y^2}{n_2}}} \sim N(0,1)$, and $I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}$.

case 1.2′ ($n_1, n_2 \geq 30$) For any independent populations $X$ and $Y$, it holds that

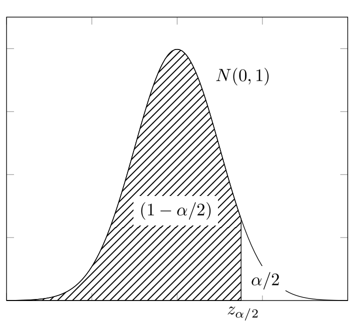
$$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1}+\frac{\sigma_Y^2}{n_2}}} \sim N(0,1) \text{ and}$$

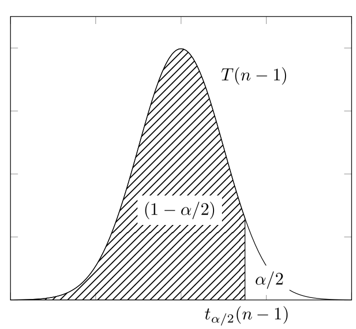
$$I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

case 1.3′ (any $n_1, n_2$) If independent populations $X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2)$, where $\sigma_X^2, \sigma_Y^2$ are unknown but $\sigma_X^2 = \sigma_Y^2$, then

$$\frac{(\bar{X}-\bar{Y})-(\mu_X-\mu_Y)}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim T(n_1+n_2-2), \text{ where } S^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}, \text{ and}$$

$$I_{\mu_X-\mu_Y} = (\bar{x}-\bar{y}) \mp t_{\alpha/2}(n_1+n_2-2) \cdot s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}.$$




## CI-2: $(1-\alpha)$ CI of population variance(s) $\sigma^2$

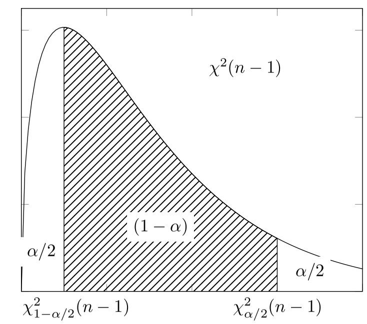
• If a population $X \sim N(\mu, \sigma^2)$ and $\sigma^2$ is unknown, then $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and

$$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}(n-1)}, \quad \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}(n-1)}\right).$$

• If two independent populations $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$, and $\sigma^2$ is unknown, then $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1+n_2-2)$, and

$$I_{\sigma^2} = \left(\frac{(n_1+n_2-2)s^2}{\chi^2_{\alpha/2}(n_1+n_2-2)}, \quad \frac{(n_1+n_2-2)s^2}{\chi^2_{1-\alpha/2}(n_1+n_2-2)}\right),$$

where $S^2 = \frac{(n_1-1)S_X^2+(n_2-1)S_Y^2}{n_1+n_2-2}$.




## CI-3: $(1-\alpha)$ CI of population proportion(s)

• If a (large) population has an unknow proportion $p$, then $\frac{\hat{P}-p}{\sqrt{p(1-p)/n}} \sim N(0,1)$ if $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$ with $\hat{p} = x/n$, and $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

• If two independent (large) populations have unknow proportions $p_1$ and $p_2$, then

$$\frac{(\hat{P}_1-\hat{P}_2)-(p_1-p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}} \sim N(0,1)$$

if $n_i\hat{p}_i \geq 10$ and $n_i(1-\hat{p}_i) \geq 10$ for $i = 1,2$, and $I_{p_1-p_2} = (\hat{p}_1-\hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

# 7. Hypothesis Test (HT)

| | $H_0$ is true | $H_0$ is false and $\theta = \theta_1$ |
|---|---|---|
| reject $H_0$ | (type I error or significance level) $\alpha$ | (power) $h(\theta_1)$ |
| don't reject $H_0$ | $1-\alpha$ | (type II error) $\beta(\theta_1) = 1 - h(\theta_1)$ |

reject $H_0 \Leftrightarrow TS \in C \Leftrightarrow p$-value$< \alpha$

### $\chi^2$ tests for populations (non-parametric tests)

Suppose that for a random sample of a population $X$, the $n$ elements of it are classified into $k$ disjoint groups $A_i, 1 \leq i \leq k$. For each group $A_i, 1 \leq i \leq k$, suppose that there are $N_i, 1 \leq i \leq k$ elements inside. Let $p_i = P(A_i)$ assuming a given distribution of $X$. Note that $p_1 + p_2 + \ldots + p_k = 1$ and $N_1 + N_2 + \ldots + N_k = n$. One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \ \ 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If $n$ is large in the sense that $np_i \geq 5$ for all $1 \leq i \leq k$, then the test statistic is

$$\sum_{i=1}^{k} \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region $C$, one can take (note that if $H_0$ is true, then $TS$ should be close to zero)

$$C = (\chi^2_\alpha(k-1),\ \infty).$$

The conclusion would be $TS \in C \iff H_0$ is rejected.

## 8. Linear and logistic regression

**(Multiple) linear regression**: $Y = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \varepsilon, \quad \varepsilon \sim N(0, \sigma^2)$.

- $Y$ : response variable (which is normal r.v.), $\{x_1, \ldots, x_k\}$ : predictors (which are scalars).
- sample: $\{(x_{11}, \ldots, x_{1k}; y_1), (x_{21}, \ldots, x_{2k}; y_2) \ldots, (x_{n1}, \ldots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$ : least square method, that is, to minimize $\sum_{i=1}^n (\hat{y}_i - y_i)^2$, where the estimated (multiple) linear regression line $\hat{y}$ is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k.$$

- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim T(n-k-1)$, this helps determine whether or not the real $\beta_j = 0$ ?
- $\sigma^2 \approx \frac{SS_E}{n-k-1}$, this gives an estimation of the size of the error.
- $R^2 = \frac{SS_R}{SS_T}$, this gives how well the model is (if $R^2 \approx 1$, then the model fits the sample very well).
- How to test $\beta_1 = \ldots = \beta_k = 0$ ? Use the random variable $\frac{SS_R/k}{SS_E/(n-k-1)} \sim F(k, n-k-1)$.

**Logistic regression**: Let $Y$ can only take 0 or 1 with $P(Y=1) = p$ and $P(Y=0) = 1-p$,

$$E(Y) = p(x_1, \ldots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k}}.$$

- $Y$ : response variable (which is Bernoulli r.v. $P(Y=1) = p$ and $P(Y=0) = 1-p$, so $E(Y) = p$), $\{x_1, \ldots, x_k\}$ : predictors (which are scalars).
- sample: $\{(x_{11}, \ldots, x_{1k}; y_1), (x_{21}, \ldots, x_{2k}; y_2) \ldots, (x_{n1}, \ldots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$ : maximal likelihood method (maximize $\prod_{i=1}^n p(x_{i1}, \ldots, x_{ik})^{y_i} (1 - p(x_{i1}, \ldots, x_{ik}))^{1-y_i}$).
- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \approx N(0,1)$ for large $n \geq 30$, this helps determine whether or not the real $\beta_j = 0$ ?
- Classification of a new object $Y(x_1, \ldots, x_k)$ as 1 or 0 according

$$Y(x_1, \ldots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \ldots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \ldots, x_k) < 0.5, \end{cases}$$

where the estimated logit function $\hat{p}(x_1, \ldots, x_k)$ is

$$\hat{p}(x_1, \ldots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \ldots + \hat{\beta}_k x_k}}.$$

# 9. Tables

(9.1) Table for $N(0,1)$ standard normal random variable $\Phi(x) = P(N(0,1) \leq x), \ x \geq 0$.
There is an important relation $\Phi(-x) = 1 - \Phi(x), \ x \geq 0$.

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 4.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

(9.2) Table for $T(f)$ random variable $F(x) = P(T(\text{f}) \leq x)$,
where f is a parameter called 'degrees of freedom'.

| | $F(x)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| f | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.9995 |
| 1 | 1.00 | 3.08 | 6.31 | 12.71 | 31.82 | 63.66 | 127.32 | 636.62 |
| 2 | 0.82 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 | 14.09 | 31.60 |
| 3 | 0.76 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 | 7.45 | 12.92 |
| 4 | 0.74 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 | 5.60 | 8.61 |
| 5 | 0.73 | 1.48 | 2.02 | 2.57 | 3.36 | 4.03 | 4.77 | 6.87 |
| 6 | 0.72 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 | 4.32 | 5.96 |
| 7 | 0.71 | 1.41 | 1.89 | 2.36 | 3.00 | 3.50 | 4.03 | 5.41 |
| 8 | 0.71 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 | 3.83 | 5.04 |
| 9 | 0.70 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 | 3.69 | 4.78 |
| 10 | 0.70 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 | 3.58 | 4.59 |
| 11 | 0.70 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 | 3.50 | 4.44 |
| 12 | 0.70 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 | 3.43 | 4.32 |
| 13 | 0.69 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 | 3.37 | 4.22 |
| 14 | 0.69 | 1.35 | 1.76 | 2.14 | 2.62 | 2.98 | 3.33 | 4.14 |
| 15 | 0.69 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 | 3.29 | 4.07 |
| 16 | 0.69 | 1.34 | 1.75 | 2.12 | 2.58 | 2.92 | 3.25 | 4.01 |
| 17 | 0.69 | 1.33 | 1.74 | 2.11 | 2.57 | 2.90 | 3.22 | 3.97 |
| 18 | 0.69 | 1.33 | 1.73 | 2.10 | 2.55 | 2.88 | 3.20 | 3.92 |
| 19 | 0.69 | 1.33 | 1.73 | 2.09 | 2.54 | 2.86 | 3.17 | 3.88 |
| 20 | 0.69 | 1.33 | 1.72 | 2.09 | 2.53 | 2.85 | 3.15 | 3.85 |
| 21 | 0.69 | 1.32 | 1.72 | 2.08 | 2.52 | 2.83 | 3.14 | 3.82 |
| 22 | 0.69 | 1.32 | 1.72 | 2.07 | 2.51 | 2.82 | 3.12 | 3.79 |
| 23 | 0.69 | 1.32 | 1.71 | 2.07 | 2.50 | 2.81 | 3.10 | 3.77 |
| 24 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.80 | 3.09 | 3.75 |
| 25 | 0.68 | 1.32 | 1.71 | 2.06 | 2.49 | 2.79 | 3.08 | 3.73 |
| 26 | 0.68 | 1.31 | 1.71 | 2.06 | 2.48 | 2.78 | 3.07 | 3.71 |
| 27 | 0.68 | 1.31 | 1.70 | 2.05 | 2.47 | 2.77 | 3.06 | 3.69 |
| 28 | 0.68 | 1.31 | 1.70 | 2.05 | 2.47 | 2.76 | 3.05 | 3.67 |
| 29 | 0.68 | 1.31 | 1.70 | 2.05 | 2.46 | 2.76 | 3.04 | 3.66 |
| 30 | 0.68 | 1.31 | 1.70 | 2.04 | 2.46 | 2.75 | 3.03 | 3.65 |
| 40 | 0.68 | 1.30 | 1.68 | 2.02 | 2.42 | 2.70 | 2.97 | 3.55 |
| 50 | 0.68 | 1.30 | 1.68 | 2.01 | 2.40 | 2.68 | 2.94 | 3.50 |
| 60 | 0.68 | 1.30 | 1.67 | 2.00 | 2.39 | 2.66 | 2.91 | 3.46 |
| 100 | 0.68 | 1.29 | 1.66 | 1.98 | 2.36 | 2.63 | 2.87 | 3.39 |
| $\infty$ | 0.67 | 1.28 | 1.65 | 1.96 | 2.33 | 2.58 | 2.81 | 3.29 |

(9.3) Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

| f | $F(x)$ 0.0005 | 0.001 | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.15 | 0.27 | 0.45 |
| 2 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.10 | 0.21 | 0.45 | 0.71 | 1.02 | 1.39 |
| 3 | 0.02 | 0.02 | 0.07 | 0.11 | 0.22 | 0.35 | 0.58 | 1.01 | 1.42 | 1.87 | 2.37 |
| 4 | 0.06 | 0.09 | 0.21 | 0.30 | 0.48 | 0.71 | 1.06 | 1.65 | 2.19 | 2.75 | 3.36 |
| 5 | 0.16 | 0.21 | 0.41 | 0.55 | 0.83 | 1.15 | 1.61 | 2.34 | 3.00 | 3.66 | 4.35 |
| 6 | 0.30 | 0.38 | 0.68 | 0.87 | 1.24 | 1.64 | 2.20 | 3.07 | 3.83 | 4.57 | 5.35 |
| 7 | 0.48 | 0.60 | 0.99 | 1.24 | 1.69 | 2.17 | 2.83 | 3.82 | 4.67 | 5.49 | 6.35 |
| 8 | 0.71 | 0.86 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 4.59 | 5.53 | 6.42 | 7.34 |
| 9 | 0.97 | 1.15 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 5.38 | 6.39 | 7.36 | 8.34 |
| 10 | 1.26 | 1.48 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 6.18 | 7.27 | 8.30 | 9.34 |
| 11 | 1.59 | 1.83 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 6.99 | 8.15 | 9.24 | 10.34 |
| 12 | 1.93 | 2.21 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 7.81 | 9.03 | 10.18 | 11.34 |
| 13 | 2.31 | 2.62 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 8.63 | 9.93 | 11.13 | 12.34 |
| 14 | 2.70 | 3.04 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 9.47 | 10.82 | 12.08 | 13.34 |
| 15 | 3.11 | 3.48 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 10.31 | 11.72 | 13.03 | 14.34 |
| 16 | 3.54 | 3.94 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 11.15 | 12.62 | 13.98 | 15.34 |
| 17 | 3.98 | 4.42 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 12.00 | 13.53 | 14.94 | 16.34 |
| 18 | 4.44 | 4.90 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 12.86 | 14.44 | 15.89 | 17.34 |
| 19 | 4.91 | 5.41 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 13.72 | 15.35 | 16.85 | 18.34 |
| 20 | 5.40 | 5.92 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 14.58 | 16.27 | 17.81 | 19.34 |
| 21 | 5.90 | 6.45 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 15.44 | 17.18 | 18.77 | 20.34 |
| 22 | 6.40 | 6.98 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 16.31 | 18.10 | 19.73 | 21.34 |
| 23 | 6.92 | 7.53 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 17.19 | 19.02 | 20.69 | 22.34 |
| 24 | 7.45 | 8.08 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 18.06 | 19.94 | 21.65 | 23.34 |
| 25 | 7.99 | 8.65 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 18.94 | 20.87 | 22.62 | 24.34 |
| 26 | 8.54 | 9.22 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 19.82 | 21.79 | 23.58 | 25.34 |
| 27 | 9.09 | 9.80 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 20.70 | 22.72 | 24.54 | 26.34 |
| 28 | 9.66 | 10.39 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 21.59 | 23.65 | 25.51 | 27.34 |
| 29 | 10.23 | 10.99 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 22.48 | 24.58 | 26.48 | 28.34 |
| 30 | 10.80 | 11.59 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 23.36 | 25.51 | 27.44 | 29.34 |
| 40 | 16.91 | 17.92 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 32.34 | 34.87 | 37.13 | 39.34 |
| 50 | 23.46 | 24.67 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 41.45 | 44.31 | 46.86 | 49.33 |
| 60 | 30.34 | 31.74 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 50.64 | 53.81 | 56.62 | 59.33 |
| 100 | 59.90 | 61.92 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 87.95 | 92.13 | 95.81 | 99.33 |

Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

| f | $F(x)$ 0.60 | 0.70 | 0.80 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.71 | 1.07 | 1.64 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.83 | 12.12 |
| 2 | 1.83 | 2.41 | 3.22 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 | 13.82 | 15.20 |
| 3 | 2.95 | 3.66 | 4.64 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 | 16.27 | 17.73 |
| 4 | 4.04 | 4.88 | 5.99 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 | 18.47 | 20.00 |
| 5 | 5.13 | 6.06 | 7.29 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 | 20.52 | 22.11 |
| 6 | 6.21 | 7.23 | 8.56 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 | 22.46 | 24.10 |
| 7 | 7.28 | 8.38 | 9.80 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 | 24.32 | 26.02 |
| 8 | 8.35 | 9.52 | 11.03 | 13.36 | 15.51 | 17.53 | 20.09 | 21.95 | 26.12 | 27.87 |
| 9 | 9.41 | 10.66 | 12.24 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 | 27.88 | 29.67 |
| 10 | 10.47 | 11.78 | 13.44 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 | 29.59 | 31.42 |
| 11 | 11.53 | 12.90 | 14.63 | 17.28 | 19.68 | 21.92 | 24.72 | 26.76 | 31.26 | 33.14 |
| 12 | 12.58 | 14.01 | 15.81 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 | 32.91 | 34.82 |
| 13 | 13.64 | 15.12 | 16.98 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 | 34.53 | 36.48 |
| 14 | 14.69 | 16.22 | 18.15 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 | 36.12 | 38.11 |
| 15 | 15.73 | 17.32 | 19.31 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 | 37.70 | 39.72 |
| 16 | 16.78 | 18.42 | 20.47 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 | 39.25 | 41.31 |
| 17 | 17.82 | 19.51 | 21.61 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 | 40.79 | 42.88 |
| 18 | 18.87 | 20.60 | 22.76 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 | 42.31 | 44.43 |
| 19 | 19.91 | 21.69 | 23.90 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 | 43.82 | 45.97 |
| 20 | 20.95 | 22.77 | 25.04 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 | 45.31 | 47.50 |
| 21 | 21.99 | 23.86 | 26.17 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 | 46.80 | 49.01 |
| 22 | 23.03 | 24.94 | 27.30 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 | 48.27 | 50.51 |
| 23 | 24.07 | 26.02 | 28.43 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 | 49.73 | 52.00 |
| 24 | 25.11 | 27.10 | 29.55 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 | 51.18 | 53.48 |
| 25 | 26.14 | 28.17 | 30.68 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 | 52.62 | 54.95 |
| 26 | 27.18 | 29.25 | 31.79 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 | 54.05 | 56.41 |
| 27 | 28.21 | 30.32 | 32.91 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 | 55.48 | 57.86 |
| 28 | 29.25 | 31.39 | 34.03 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 | 56.89 | 59.30 |
| 29 | 30.28 | 32.46 | 35.14 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 | 58.30 | 60.73 |
| 30 | 31.32 | 33.53 | 36.25 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 | 59.70 | 62.16 |
| 40 | 41.62 | 44.16 | 47.27 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 | 73.40 | 76.09 |
| 50 | 51.89 | 54.72 | 58.16 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 | 86.66 | 89.56 |
| 60 | 62.13 | 65.23 | 68.97 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 | 99.61 | 102.69 |
| 100 | 102.95 | 106.91 | 111.67 | 118.50 | 124.34 | 129.56 | 135.81 | 140.17 | 149.45 | 153.17 |

(9.4) Table for Binomial random variable $P(Bin(n,p) \le k)$ if $p \le 0.5$.
If $p > 0.5$, then $P(Bin(n,p) \le k) = P(Bin(n, 1-p) \ge n-k)$.

| $n$ | $k$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
| | 1 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8775 | 0.8400 | 0.7975 | 0.7500 |
| 3 | 0 | 0.8574 | 0.7290 | 0.6141 | 0.5120 | 0.4219 | 0.3430 | 0.2746 | 0.2160 | 0.1664 | 0.1250 |
| | 1 | 0.9928 | 0.9720 | 0.9392 | 0.8960 | 0.8438 | 0.7840 | 0.7183 | 0.6480 | 0.5747 | 0.5000 |
| | 2 | 0.9999 | 0.9990 | 0.9966 | 0.9920 | 0.9844 | 0.9730 | 0.9571 | 0.9360 | 0.9089 | 0.8750 |
| 4 | 0 | 0.8145 | 0.6561 | 0.5220 | 0.4096 | 0.3164 | 0.2401 | 0.1785 | 0.1296 | 0.0915 | 0.0625 |
| | 1 | 0.9860 | 0.9477 | 0.8905 | 0.8192 | 0.7383 | 0.6517 | 0.5630 | 0.4752 | 0.3910 | 0.3125 |
| | 2 | 0.9995 | 0.9963 | 0.9880 | 0.9728 | 0.9492 | 0.9163 | 0.8735 | 0.8208 | 0.7585 | 0.6875 |
| | 3 | 1.0000 | 0.9999 | 0.9995 | 0.9984 | 0.9961 | 0.9919 | 0.9850 | 0.9744 | 0.9590 | 0.9375 |
| 5 | 0 | 0.7738 | 0.5905 | 0.4437 | 0.3277 | 0.2373 | 0.1681 | 0.1160 | 0.0778 | 0.0503 | 0.0313 |
| | 1 | 0.9774 | 0.9185 | 0.8352 | 0.7373 | 0.6328 | 0.5282 | 0.4284 | 0.3370 | 0.2562 | 0.1875 |
| | 2 | 0.9988 | 0.9914 | 0.9734 | 0.9421 | 0.8965 | 0.8369 | 0.7648 | 0.6826 | 0.5931 | 0.5000 |
| | 3 | 1.0000 | 0.9995 | 0.9978 | 0.9933 | 0.9844 | 0.9692 | 0.9460 | 0.9130 | 0.8688 | 0.8125 |
| | 4 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9947 | 0.9898 | 0.9815 | 0.9688 |
| 6 | 0 | 0.7351 | 0.5314 | 0.3771 | 0.2621 | 0.1780 | 0.1176 | 0.0754 | 0.0467 | 0.0277 | 0.0156 |
| | 1 | 0.9672 | 0.8857 | 0.7765 | 0.6554 | 0.5339 | 0.4202 | 0.3191 | 0.2333 | 0.1636 | 0.1094 |
| | 2 | 0.9978 | 0.9842 | 0.9527 | 0.9011 | 0.8306 | 0.7443 | 0.6471 | 0.5443 | 0.4415 | 0.3438 |
| | 3 | 0.9999 | 0.9987 | 0.9941 | 0.9830 | 0.9624 | 0.9295 | 0.8826 | 0.8208 | 0.7447 | 0.6563 |
| | 4 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9954 | 0.9891 | 0.9777 | 0.9590 | 0.9308 | 0.8906 |
| | 5 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9982 | 0.9959 | 0.9917 | 0.9844 |
| 7 | 0 | 0.6983 | 0.4783 | 0.3206 | 0.2097 | 0.1335 | 0.0824 | 0.0490 | 0.0280 | 0.0152 | 0.0078 |
| | 1 | 0.9556 | 0.8503 | 0.7166 | 0.5767 | 0.4449 | 0.3294 | 0.2338 | 0.1586 | 0.1024 | 0.0625 |
| | 2 | 0.9962 | 0.9743 | 0.9262 | 0.8520 | 0.7564 | 0.6471 | 0.5323 | 0.4199 | 0.3164 | 0.2266 |
| | 3 | 0.9998 | 0.9973 | 0.9879 | 0.9667 | 0.9294 | 0.8740 | 0.8002 | 0.7102 | 0.6083 | 0.5000 |
| | 4 | 1.0000 | 0.9998 | 0.9988 | 0.9953 | 0.9871 | 0.9712 | 0.9444 | 0.9037 | 0.8471 | 0.7734 |
| | 5 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9910 | 0.9812 | 0.9643 | 0.9375 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9984 | 0.9963 | 0.9922 |
| 8 | 0 | 0.6634 | 0.4305 | 0.2725 | 0.1678 | 0.1001 | 0.0576 | 0.0319 | 0.0168 | 0.0084 | 0.0039 |
| | 1 | 0.9428 | 0.8131 | 0.6572 | 0.5033 | 0.3671 | 0.2553 | 0.1691 | 0.1064 | 0.0632 | 0.0352 |
| | 2 | 0.9942 | 0.9619 | 0.8948 | 0.7969 | 0.6785 | 0.5518 | 0.4278 | 0.3154 | 0.2201 | 0.1445 |
| | 3 | 0.9996 | 0.9950 | 0.9786 | 0.9437 | 0.8862 | 0.8059 | 0.7064 | 0.5941 | 0.4770 | 0.3633 |
| | 4 | 1.0000 | 0.9996 | 0.9971 | 0.9896 | 0.9727 | 0.9420 | 0.8939 | 0.8263 | 0.7396 | 0.6367 |
| | 5 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9887 | 0.9747 | 0.9502 | 0.9115 | 0.8555 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9964 | 0.9915 | 0.9819 | 0.9648 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9983 | 0.9961 |
| 9 | 0 | 0.6302 | 0.3874 | 0.2316 | 0.1342 | 0.0751 | 0.0404 | 0.0207 | 0.0101 | 0.0046 | 0.0020 |
| | 1 | 0.9288 | 0.7748 | 0.5995 | 0.4362 | 0.3003 | 0.1960 | 0.1211 | 0.0705 | 0.0385 | 0.0195 |
| | 2 | 0.9916 | 0.9470 | 0.8591 | 0.7382 | 0.6007 | 0.4628 | 0.3373 | 0.2318 | 0.1495 | 0.0898 |
| | 3 | 0.9994 | 0.9917 | 0.9661 | 0.9144 | 0.8343 | 0.7297 | 0.6089 | 0.4826 | 0.3614 | 0.2539 |
| | 4 | 1.0000 | 0.9991 | 0.9944 | 0.9804 | 0.9511 | 0.9012 | 0.8283 | 0.7334 | 0.6214 | 0.5000 |
| | 5 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9900 | 0.9747 | 0.9464 | 0.9006 | 0.8342 | 0.7461 |
| | 6 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9957 | 0.9888 | 0.9750 | 0.9502 | 0.9102 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9986 | 0.9962 | 0.9909 | 0.9805 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9980 |

Table for Binomial random variable $P(Bin(n,p) \le k)$ if $p \le 0.5$.
If $p > 0.5$, then $P(Bin(n,p) \le k) = P(Bin(n, 1-p) \ge n-k)$.

| $n$ | $k$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0 | 0.5987 | 0.3487 | 0.1969 | 0.1074 | 0.0563 | 0.0282 | 0.0135 | 0.0060 | 0.0025 | 0.0010 |
| | 1 | 0.9139 | 0.7361 | 0.5443 | 0.3758 | 0.2440 | 0.1493 | 0.0860 | 0.0464 | 0.0233 | 0.0107 |
| | 2 | 0.9885 | 0.9298 | 0.8202 | 0.6778 | 0.5256 | 0.3828 | 0.2616 | 0.1673 | 0.0996 | 0.0547 |
| | 3 | 0.9990 | 0.9872 | 0.9500 | 0.8791 | 0.7759 | 0.6496 | 0.5138 | 0.3823 | 0.2660 | 0.1719 |
| | 4 | 0.9999 | 0.9984 | 0.9901 | 0.9672 | 0.9219 | 0.8497 | 0.7515 | 0.6331 | 0.5044 | 0.3770 |
| | 5 | 1.0000 | 0.9999 | 0.9986 | 0.9936 | 0.9803 | 0.9527 | 0.9051 | 0.8338 | 0.7384 | 0.6230 |
| | 6 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9965 | 0.9894 | 0.9740 | 0.9452 | 0.8980 | 0.8281 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9952 | 0.9877 | 0.9726 | 0.9453 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9983 | 0.9955 | 0.9893 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 |
| 11 | 0 | 0.5688 | 0.3138 | 0.1673 | 0.0859 | 0.0422 | 0.0198 | 0.0088 | 0.0036 | 0.0014 | 0.0005 |
| | 1 | 0.8981 | 0.6974 | 0.4922 | 0.3221 | 0.1971 | 0.1130 | 0.0606 | 0.0302 | 0.0139 | 0.0059 |
| | 2 | 0.9848 | 0.9104 | 0.7788 | 0.6174 | 0.4552 | 0.3127 | 0.2001 | 0.1189 | 0.0652 | 0.0327 |
| | 3 | 0.9984 | 0.9815 | 0.9306 | 0.8389 | 0.7133 | 0.5696 | 0.4256 | 0.2963 | 0.1911 | 0.1133 |
| | 4 | 0.9999 | 0.9972 | 0.9841 | 0.9496 | 0.8854 | 0.7897 | 0.6683 | 0.5328 | 0.3971 | 0.2744 |
| | 5 | 1.0000 | 0.9997 | 0.9973 | 0.9883 | 0.9657 | 0.9218 | 0.8513 | 0.7535 | 0.6331 | 0.5000 |
| | 6 | 1.0000 | 1.0000 | 0.9997 | 0.9980 | 0.9924 | 0.9784 | 0.9499 | 0.9006 | 0.8262 | 0.7256 |
| | 7 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9957 | 0.9878 | 0.9707 | 0.9390 | 0.8867 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9980 | 0.9941 | 0.9852 | 0.9673 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9993 | 0.9978 | 0.9941 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9995 |
| 12 | 0 | 0.5404 | 0.2824 | 0.1422 | 0.0687 | 0.0317 | 0.0138 | 0.0057 | 0.0022 | 0.0008 | 0.0002 |
| | 1 | 0.8816 | 0.6590 | 0.4435 | 0.2749 | 0.1584 | 0.0850 | 0.0424 | 0.0196 | 0.0083 | 0.0032 |
| | 2 | 0.9804 | 0.8891 | 0.7358 | 0.5583 | 0.3907 | 0.2528 | 0.1513 | 0.0834 | 0.0421 | 0.0193 |
| | 3 | 0.9978 | 0.9744 | 0.9078 | 0.7946 | 0.6488 | 0.4925 | 0.3467 | 0.2253 | 0.1345 | 0.0730 |
| | 4 | 0.9998 | 0.9957 | 0.9761 | 0.9274 | 0.8424 | 0.7237 | 0.5833 | 0.4382 | 0.3044 | 0.1938 |
| | 5 | 1.0000 | 0.9995 | 0.9954 | 0.9806 | 0.9456 | 0.8822 | 0.7873 | 0.6652 | 0.5269 | 0.3872 |
| | 6 | 1.0000 | 0.9999 | 0.9993 | 0.9961 | 0.9857 | 0.9614 | 0.9154 | 0.8418 | 0.7393 | 0.6128 |
| | 7 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9972 | 0.9905 | 0.9745 | 0.9427 | 0.8883 | 0.8062 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9983 | 0.9944 | 0.9847 | 0.9644 | 0.9270 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9992 | 0.9972 | 0.9921 | 0.9807 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9968 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 13 | 0 | 0.5133 | 0.2542 | 0.1209 | 0.0550 | 0.0238 | 0.0097 | 0.0037 | 0.0013 | 0.0004 | 0.0001 |
| | 1 | 0.8646 | 0.6213 | 0.3983 | 0.2336 | 0.1267 | 0.0637 | 0.0296 | 0.0126 | 0.0049 | 0.0017 |
| | 2 | 0.9755 | 0.8661 | 0.6920 | 0.5017 | 0.3326 | 0.2025 | 0.1132 | 0.0579 | 0.0269 | 0.0112 |
| | 3 | 0.9969 | 0.9658 | 0.8820 | 0.7473 | 0.5843 | 0.4206 | 0.2783 | 0.1686 | 0.0929 | 0.0461 |
| | 4 | 0.9997 | 0.9935 | 0.9658 | 0.9009 | 0.7940 | 0.6543 | 0.5005 | 0.3530 | 0.2279 | 0.1334 |
| | 5 | 1.0000 | 0.9991 | 0.9925 | 0.9700 | 0.9198 | 0.8346 | 0.7159 | 0.5744 | 0.4268 | 0.2905 |
| | 6 | 1.0000 | 0.9999 | 0.9987 | 0.9930 | 0.9757 | 0.9376 | 0.8705 | 0.7712 | 0.6437 | 0.5000 |
| | 7 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9944 | 0.9818 | 0.9538 | 0.9023 | 0.8212 | 0.7095 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9990 | 0.9960 | 0.9874 | 0.9679 | 0.9302 | 0.8666 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9975 | 0.9922 | 0.9797 | 0.9539 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9987 | 0.9959 | 0.9888 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9983 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |

Table for Binomial random variable $P(Bin(n,p) \le k)$ if $p \le 0.5$.
If $p > 0.5$, then $P(Bin(n,p) \le k) = P(Bin(n, 1-p) \ge n-k)$.

| $n$ | $k$ | $p$ = 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 0 | 0.4877 | 0.2288 | 0.1028 | 0.0440 | 0.0178 | 0.0068 | 0.0024 | 0.0008 | 0.0002 | 0.0001 |
| | 1 | 0.8470 | 0.5846 | 0.3567 | 0.1979 | 0.1010 | 0.0475 | 0.0205 | 0.0081 | 0.0029 | 0.0009 |
| | 2 | 0.9699 | 0.8416 | 0.6479 | 0.4481 | 0.2811 | 0.1608 | 0.0839 | 0.0398 | 0.0170 | 0.0065 |
| | 3 | 0.9958 | 0.9559 | 0.8535 | 0.6982 | 0.5213 | 0.3552 | 0.2205 | 0.1243 | 0.0632 | 0.0287 |
| | 4 | 0.9996 | 0.9908 | 0.9533 | 0.8702 | 0.7415 | 0.5842 | 0.4227 | 0.2793 | 0.1672 | 0.0898 |
| | 5 | 1.0000 | 0.9985 | 0.9885 | 0.9561 | 0.8883 | 0.7805 | 0.6405 | 0.4859 | 0.3373 | 0.2120 |
| | 6 | 1.0000 | 0.9998 | 0.9978 | 0.9884 | 0.9617 | 0.9067 | 0.8164 | 0.6925 | 0.5461 | 0.3953 |
| | 7 | 1.0000 | 1.0000 | 0.9997 | 0.9976 | 0.9897 | 0.9685 | 0.9247 | 0.8499 | 0.7414 | 0.6047 |
| | 8 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9978 | 0.9917 | 0.9757 | 0.9417 | 0.8811 | 0.7880 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9983 | 0.9940 | 0.9825 | 0.9574 | 0.9102 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9989 | 0.9961 | 0.9886 | 0.9713 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9978 | 0.9935 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 15 | 0 | 0.4633 | 0.2059 | 0.0874 | 0.0352 | 0.0134 | 0.0047 | 0.0016 | 0.0005 | 0.0001 | 0.0000 |
| | 1 | 0.8290 | 0.5490 | 0.3186 | 0.1671 | 0.0802 | 0.0353 | 0.0142 | 0.0052 | 0.0017 | 0.0005 |
| | 2 | 0.9638 | 0.8159 | 0.6042 | 0.3980 | 0.2361 | 0.1268 | 0.0617 | 0.0271 | 0.0107 | 0.0037 |
| | 3 | 0.9945 | 0.9444 | 0.8227 | 0.6482 | 0.4613 | 0.2969 | 0.1727 | 0.0905 | 0.0424 | 0.0176 |
| | 4 | 0.9994 | 0.9873 | 0.9383 | 0.8358 | 0.6865 | 0.5155 | 0.3519 | 0.2173 | 0.1204 | 0.0592 |
| | 5 | 0.9999 | 0.9978 | 0.9832 | 0.9389 | 0.8516 | 0.7216 | 0.5643 | 0.4032 | 0.2608 | 0.1509 |
| | 6 | 1.0000 | 0.9997 | 0.9964 | 0.9819 | 0.9434 | 0.8689 | 0.7548 | 0.6098 | 0.4522 | 0.3036 |
| | 7 | 1.0000 | 1.0000 | 0.9994 | 0.9958 | 0.9827 | 0.9500 | 0.8868 | 0.7869 | 0.6535 | 0.5000 |
| | 8 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9958 | 0.9848 | 0.9578 | 0.9050 | 0.8182 | 0.6964 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9963 | 0.9876 | 0.9662 | 0.9231 | 0.8491 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9972 | 0.9907 | 0.9745 | 0.9408 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9981 | 0.9937 | 0.9824 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9963 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 16 | 0 | 0.4401 | 0.1853 | 0.0743 | 0.0281 | 0.0100 | 0.0033 | 0.0010 | 0.0003 | 0.0001 | 0.0000 |
| | 1 | 0.8108 | 0.5147 | 0.2839 | 0.1407 | 0.0635 | 0.0261 | 0.0098 | 0.0033 | 0.0010 | 0.0003 |
| | 2 | 0.9571 | 0.7892 | 0.5614 | 0.3518 | 0.1971 | 0.0994 | 0.0451 | 0.0183 | 0.0066 | 0.0021 |
| | 3 | 0.9930 | 0.9316 | 0.7899 | 0.5981 | 0.4050 | 0.2459 | 0.1339 | 0.0651 | 0.0281 | 0.0106 |
| | 4 | 0.9991 | 0.9830 | 0.9209 | 0.7982 | 0.6302 | 0.4499 | 0.2892 | 0.1666 | 0.0853 | 0.0384 |
| | 5 | 0.9999 | 0.9967 | 0.9765 | 0.9183 | 0.8103 | 0.6598 | 0.4900 | 0.3288 | 0.1976 | 0.1051 |
| | 6 | 1.0000 | 0.9995 | 0.9944 | 0.9733 | 0.9204 | 0.8247 | 0.6881 | 0.5272 | 0.3660 | 0.2272 |
| | 7 | 1.0000 | 0.9999 | 0.9989 | 0.9930 | 0.9729 | 0.9256 | 0.8406 | 0.7161 | 0.5629 | 0.4018 |
| | 8 | 1.0000 | 1.0000 | 0.9998 | 0.9985 | 0.9925 | 0.9743 | 0.9329 | 0.8577 | 0.7441 | 0.5982 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9984 | 0.9929 | 0.9771 | 0.9417 | 0.8759 | 0.7728 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9938 | 0.9809 | 0.9514 | 0.8949 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9951 | 0.9851 | 0.9616 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9991 | 0.9965 | 0.9894 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9979 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Table for Binomial random variable $P(Bin(n,p) \le k)$ if $p \le 0.5$.
If $p > 0.5$, then $P(Bin(n,p) \le k) = P(Bin(n, 1-p) \ge n-k)$.

| $n$ | $k$ | $p$ = 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 0 | 0.4181 | 0.1668 | 0.0631 | 0.0225 | 0.0075 | 0.0023 | 0.0007 | 0.0002 | 0.0000 | 0.0000 |
| | 1 | 0.7922 | 0.4818 | 0.2525 | 0.1182 | 0.0501 | 0.0193 | 0.0067 | 0.0021 | 0.0006 | 0.0001 |
| | 2 | 0.9497 | 0.7618 | 0.5198 | 0.3096 | 0.1637 | 0.0774 | 0.0327 | 0.0123 | 0.0041 | 0.0012 |
| | 3 | 0.9912 | 0.9174 | 0.7556 | 0.5489 | 0.3530 | 0.2019 | 0.1028 | 0.0464 | 0.0184 | 0.0064 |
| | 4 | 0.9988 | 0.9779 | 0.9013 | 0.7582 | 0.5739 | 0.3887 | 0.2348 | 0.1260 | 0.0596 | 0.0245 |
| | 5 | 0.9999 | 0.9953 | 0.9681 | 0.8943 | 0.7653 | 0.5968 | 0.4197 | 0.2639 | 0.1471 | 0.0717 |
| | 6 | 1.0000 | 0.9992 | 0.9917 | 0.9623 | 0.8929 | 0.7752 | 0.6188 | 0.4478 | 0.2902 | 0.1662 |
| | 7 | 1.0000 | 0.9999 | 0.9983 | 0.9891 | 0.9598 | 0.8954 | 0.7872 | 0.6405 | 0.4743 | 0.3145 |
| | 8 | 1.0000 | 1.0000 | 0.9997 | 0.9974 | 0.9876 | 0.9597 | 0.9006 | 0.8011 | 0.6626 | 0.5000 |
| | 9 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9969 | 0.9873 | 0.9617 | 0.9081 | 0.8166 | 0.6855 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9968 | 0.9880 | 0.9652 | 0.9174 | 0.8338 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9970 | 0.9894 | 0.9699 | 0.9283 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9975 | 0.9914 | 0.9755 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9981 | 0.9936 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9988 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 18 | 0 | 0.3972 | 0.1501 | 0.0536 | 0.0180 | 0.0056 | 0.0016 | 0.0004 | 0.0001 | 0.0000 | 0.0000 |
| | 1 | 0.7735 | 0.4503 | 0.2241 | 0.0991 | 0.0395 | 0.0142 | 0.0046 | 0.0013 | 0.0003 | 0.0001 |
| | 2 | 0.9419 | 0.7338 | 0.4797 | 0.2713 | 0.1353 | 0.0600 | 0.0236 | 0.0082 | 0.0025 | 0.0007 |
| | 3 | 0.9891 | 0.9018 | 0.7202 | 0.5010 | 0.3057 | 0.1646 | 0.0783 | 0.0328 | 0.0120 | 0.0038 |
| | 4 | 0.9985 | 0.9718 | 0.8794 | 0.7164 | 0.5187 | 0.3327 | 0.1886 | 0.0942 | 0.0411 | 0.0154 |
| | 5 | 0.9998 | 0.9936 | 0.9581 | 0.8671 | 0.7175 | 0.5344 | 0.3550 | 0.2088 | 0.1077 | 0.0481 |
| | 6 | 1.0000 | 0.9988 | 0.9882 | 0.9487 | 0.8610 | 0.7217 | 0.5491 | 0.3743 | 0.2258 | 0.1189 |
| | 7 | 1.0000 | 0.9998 | 0.9973 | 0.9837 | 0.9431 | 0.8593 | 0.7283 | 0.5634 | 0.3915 | 0.2403 |
| | 8 | 1.0000 | 1.0000 | 0.9995 | 0.9957 | 0.9807 | 0.9404 | 0.8609 | 0.7368 | 0.5778 | 0.4073 |
| | 9 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9946 | 0.9790 | 0.9403 | 0.8653 | 0.7473 | 0.5927 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9939 | 0.9788 | 0.9424 | 0.8720 | 0.7597 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9986 | 0.9938 | 0.9797 | 0.9463 | 0.8811 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9986 | 0.9942 | 0.9817 | 0.9519 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9951 | 0.9846 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9990 | 0.9962 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 19 | 0 | 0.3774 | 0.1351 | 0.0456 | 0.0144 | 0.0042 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| | 1 | 0.7547 | 0.4203 | 0.1985 | 0.0829 | 0.0310 | 0.0104 | 0.0031 | 0.0008 | 0.0002 | 0.0000 |
| | 2 | 0.9335 | 0.7054 | 0.4413 | 0.2369 | 0.1113 | 0.0462 | 0.0170 | 0.0055 | 0.0015 | 0.0004 |
| | 3 | 0.9868 | 0.8850 | 0.6841 | 0.4551 | 0.2631 | 0.1332 | 0.0591 | 0.0230 | 0.0077 | 0.0022 |
| | 4 | 0.9980 | 0.9648 | 0.8556 | 0.6733 | 0.4654 | 0.2822 | 0.1500 | 0.0696 | 0.0280 | 0.0096 |
| | 5 | 0.9998 | 0.9914 | 0.9463 | 0.8369 | 0.6678 | 0.4739 | 0.2968 | 0.1629 | 0.0777 | 0.0318 |
| | 6 | 1.0000 | 0.9983 | 0.9837 | 0.9324 | 0.8251 | 0.6655 | 0.4812 | 0.3081 | 0.1727 | 0.0835 |
| | 7 | 1.0000 | 0.9997 | 0.9959 | 0.9767 | 0.9225 | 0.8180 | 0.6656 | 0.4878 | 0.3169 | 0.1796 |
| | 8 | 1.0000 | 1.0000 | 0.9992 | 0.9933 | 0.9713 | 0.9161 | 0.8145 | 0.6675 | 0.4940 | 0.3238 |
| | 9 | 1.0000 | 1.0000 | 0.9999 | 0.9984 | 0.9911 | 0.9674 | 0.9125 | 0.8139 | 0.6710 | 0.5000 |
| | 10 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9977 | 0.9895 | 0.9653 | 0.9115 | 0.8159 | 0.6762 |
| | 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9972 | 0.9886 | 0.9648 | 0.9129 | 0.8204 |
| | 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9969 | 0.9884 | 0.9658 | 0.9165 |
| | 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9969 | 0.9891 | 0.9682 |
| | 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9972 | 0.9904 |
| | 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9978 |
| | 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 |

(9.5) Table for Poisson random variable $P(Po(\mu) \le k)$.

| $k$ \ $\mu$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |
| 1 | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 | 0.7358 |
| 2 | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 | 0.9197 |
| 3 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 | 0.9810 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 | 0.9963 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 | 0.9994 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $k$ \ $\mu$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 | 0.1353 |
| 1 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 | 0.4060 |
| 2 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 | 0.6767 |
| 3 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 | 0.8571 |
| 4 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 | 0.9473 |
| 5 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 | 0.9834 |
| 6 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 | 0.9955 |
| 7 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 | 0.9989 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $k$ \ $\mu$ | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 | 0.0498 |
| 1 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 | 0.1991 |
| 2 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 | 0.4232 |
| 3 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 | 0.6472 |
| 4 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 | 0.8153 |
| 5 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 | 0.9161 |
| 6 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 | 0.9665 |
| 7 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 | 0.9881 |
| 8 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 | 0.9962 |
| 9 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 | 0.9989 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Table for Poisson random variable $P(Po(\mu) \le k)$.

| $k$ \ $\mu$ | 3.2 | 3.4 | 3.6 | 3.8 | 4.0 | 4.2 | 4.4 | 4.6 | 4.8 | 5.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.0408 | 0.0334 | 0.0273 | 0.0224 | 0.0183 | 0.0150 | 0.0123 | 0.0101 | 0.0082 | 0.0067 |
| 1 | 0.1712 | 0.1468 | 0.1257 | 0.1074 | 0.0916 | 0.0780 | 0.0663 | 0.0563 | 0.0477 | 0.0404 |
| 2 | 0.3799 | 0.3397 | 0.3027 | 0.2689 | 0.2381 | 0.2102 | 0.1851 | 0.1626 | 0.1425 | 0.1247 |
| 3 | 0.6025 | 0.5584 | 0.5152 | 0.4735 | 0.4335 | 0.3954 | 0.3594 | 0.3257 | 0.2942 | 0.2650 |
| 4 | 0.7806 | 0.7442 | 0.7064 | 0.6678 | 0.6288 | 0.5898 | 0.5512 | 0.5132 | 0.4763 | 0.4405 |
| 5 | 0.8946 | 0.8705 | 0.8441 | 0.8156 | 0.7851 | 0.7531 | 0.7199 | 0.6858 | 0.6510 | 0.6160 |
| 6 | 0.9554 | 0.9421 | 0.9267 | 0.9091 | 0.8893 | 0.8675 | 0.8436 | 0.8180 | 0.7908 | 0.7622 |
| 7 | 0.9832 | 0.9769 | 0.9692 | 0.9599 | 0.9489 | 0.9361 | 0.9214 | 0.9049 | 0.8867 | 0.8666 |
| 8 | 0.9943 | 0.9917 | 0.9883 | 0.9840 | 0.9786 | 0.9721 | 0.9642 | 0.9549 | 0.9442 | 0.9319 |
| 9 | 0.9982 | 0.9973 | 0.9960 | 0.9942 | 0.9919 | 0.9889 | 0.9851 | 0.9805 | 0.9749 | 0.9682 |
| 10 | 0.9995 | 0.9992 | 0.9987 | 0.9981 | 0.9972 | 0.9959 | 0.9943 | 0.9922 | 0.9896 | 0.9863 |
| 11 | 0.9999 | 0.9998 | 0.9996 | 0.9994 | 0.9991 | 0.9986 | 0.9980 | 0.9971 | 0.9960 | 0.9945 |
| 12 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9993 | 0.9990 | 0.9986 | 0.9980 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9995 | 0.9993 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |

| $k$ \ $\mu$ | 5.2 | 5.4 | 5.6 | 5.8 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.0055 | 0.0045 | 0.0037 | 0.0030 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 |
| 1 | 0.0342 | 0.0289 | 0.0244 | 0.0206 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 |
| 2 | 0.1088 | 0.0948 | 0.0824 | 0.0715 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 |
| 3 | 0.2381 | 0.2133 | 0.1906 | 0.1700 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 |
| 4 | 0.4061 | 0.3733 | 0.3422 | 0.3127 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 |
| 5 | 0.5809 | 0.5461 | 0.5119 | 0.4783 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 |
| 6 | 0.7324 | 0.7017 | 0.6703 | 0.6384 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 |
| 7 | 0.8449 | 0.8217 | 0.7970 | 0.7710 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 |
| 8 | 0.9181 | 0.9027 | 0.8857 | 0.8672 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 |
| 9 | 0.9603 | 0.9512 | 0.9409 | 0.9292 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 |
| 10 | 0.9823 | 0.9775 | 0.9718 | 0.9651 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 |
| 11 | 0.9927 | 0.9904 | 0.9875 | 0.9841 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 |
| 12 | 0.9972 | 0.9962 | 0.9949 | 0.9932 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 |
| 13 | 0.9990 | 0.9986 | 0.9980 | 0.9973 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 |
| 14 | 0.9997 | 0.9995 | 0.9993 | 0.9990 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 |
| 15 | 0.9999 | 0.9998 | 0.9998 | 0.9996 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 |
| 16 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Table for Poisson random variable $P(Po(\mu) \leq k)$.

| | $\mu$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $k$ | 9.0 | 9.5 | 10.0 | 11.0 | 12.0 | 13.0 | 14.0 | 15.0 | 16.0 | 17.0 |
| 0 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0012 | 0.0008 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0062 | 0.0042 | 0.0028 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0212 | 0.0149 | 0.0103 | 0.0049 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 |
| 4 | 0.0550 | 0.0403 | 0.0293 | 0.0151 | 0.0076 | 0.0037 | 0.0018 | 0.0009 | 0.0004 | 0.0002 |
| 5 | 0.1157 | 0.0885 | 0.0671 | 0.0375 | 0.0203 | 0.0107 | 0.0055 | 0.0028 | 0.0014 | 0.0007 |
| 6 | 0.2068 | 0.1649 | 0.1301 | 0.0786 | 0.0458 | 0.0259 | 0.0142 | 0.0076 | 0.0040 | 0.0021 |
| 7 | 0.3239 | 0.2687 | 0.2202 | 0.1432 | 0.0895 | 0.0540 | 0.0316 | 0.0180 | 0.0100 | 0.0054 |
| 8 | 0.4557 | 0.3918 | 0.3328 | 0.2320 | 0.1550 | 0.0998 | 0.0621 | 0.0374 | 0.0220 | 0.0126 |
| 9 | 0.5874 | 0.5218 | 0.4579 | 0.3405 | 0.2424 | 0.1658 | 0.1094 | 0.0699 | 0.0433 | 0.0261 |
| 10 | 0.7060 | 0.6453 | 0.5830 | 0.4599 | 0.3472 | 0.2517 | 0.1757 | 0.1185 | 0.0774 | 0.0491 |
| 11 | 0.8030 | 0.7520 | 0.6968 | 0.5793 | 0.4616 | 0.3532 | 0.2600 | 0.1848 | 0.1270 | 0.0847 |
| 12 | 0.8758 | 0.8364 | 0.7916 | 0.6887 | 0.5760 | 0.4631 | 0.3585 | 0.2676 | 0.1931 | 0.1350 |
| 13 | 0.9261 | 0.8981 | 0.8645 | 0.7813 | 0.6815 | 0.5730 | 0.4644 | 0.3632 | 0.2745 | 0.2009 |
| 14 | 0.9585 | 0.9400 | 0.9165 | 0.8540 | 0.7720 | 0.6751 | 0.5704 | 0.4657 | 0.3675 | 0.2808 |
| 15 | 0.9780 | 0.9665 | 0.9513 | 0.9074 | 0.8444 | 0.7636 | 0.6694 | 0.5681 | 0.4667 | 0.3715 |
| 16 | 0.9889 | 0.9823 | 0.9730 | 0.9441 | 0.8987 | 0.8355 | 0.7559 | 0.6641 | 0.5660 | 0.4677 |
| 17 | 0.9947 | 0.9911 | 0.9857 | 0.9678 | 0.9370 | 0.8905 | 0.8272 | 0.7489 | 0.6593 | 0.5640 |
| 18 | 0.9976 | 0.9957 | 0.9928 | 0.9823 | 0.9626 | 0.9302 | 0.8826 | 0.8195 | 0.7423 | 0.6550 |
| 19 | 0.9989 | 0.9980 | 0.9965 | 0.9907 | 0.9787 | 0.9573 | 0.9235 | 0.8752 | 0.8122 | 0.7363 |
| 20 | 0.9996 | 0.9991 | 0.9984 | 0.9953 | 0.9884 | 0.9750 | 0.9521 | 0.9170 | 0.8682 | 0.8055 |
| 21 | 0.9998 | 0.9996 | 0.9993 | 0.9977 | 0.9939 | 0.9859 | 0.9712 | 0.9469 | 0.9108 | 0.8615 |
| 22 | 0.9999 | 0.9999 | 0.9997 | 0.9990 | 0.9970 | 0.9924 | 0.9833 | 0.9673 | 0.9418 | 0.9047 |
| 23 | 1.0000 | 0.9999 | 0.9999 | 0.9995 | 0.9985 | 0.9960 | 0.9907 | 0.9805 | 0.9633 | 0.9367 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9993 | 0.9980 | 0.9950 | 0.9888 | 0.9777 | 0.9594 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9974 | 0.9938 | 0.9869 | 0.9748 |
| 26 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9987 | 0.9967 | 0.9925 | 0.9848 |
| 27 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9983 | 0.9959 | 0.9912 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9978 | 0.9950 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9989 | 0.9973 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9986 |
| 31 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9993 |
| 32 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 |
| 33 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 34 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 35 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |