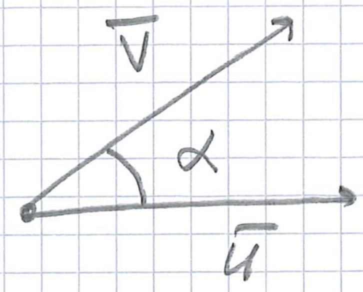
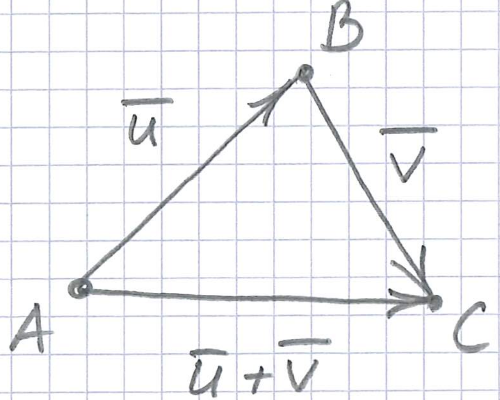
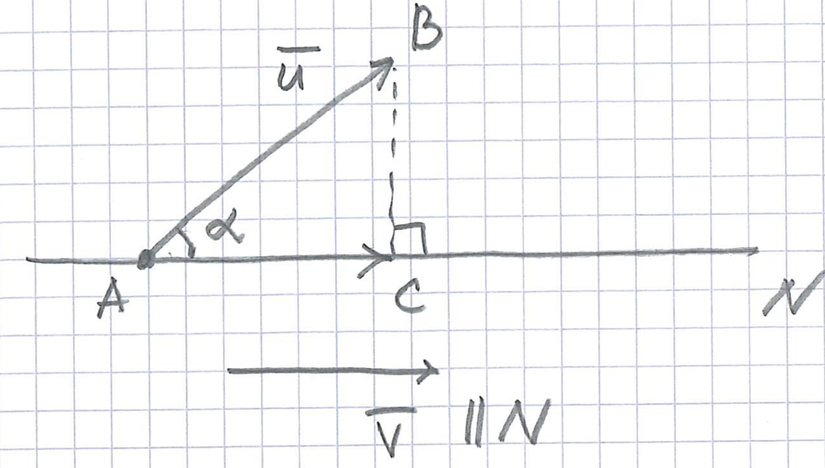


$$\bar{u} \cdot \bar{v} = |\bar{u}| \cdot |\bar{v}| \cdot \cos\alpha$$



$$|\bar{u} + \bar{v}| \leq |\bar{u}| + |\bar{v}|$$



- $$pr_N \bar{u} = \frac{\bar{u}\cdot\bar{v}}{|\bar{v}|^2}\cdot\bar{v}$$

$$\overline{AC} = |\overline{AC}|\cdot\left(\frac{1}{|\bar{v}|}\cdot\bar{v}\right) =$$

enhetsvektor

$$= \frac{|\overline{AB}|\cdot\cos\alpha\cdot|\bar{v}|}{|\bar{v}|}\cdot\frac{1}{|\bar{v}|}\cdot\bar{v} =$$

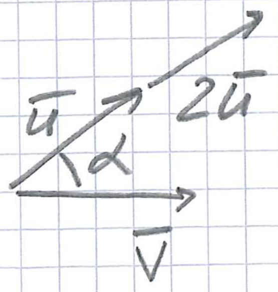
$$= \frac{\bar{u}\cdot\bar{v}}{|\bar{v}|^2}\cdot\bar{v}$$ vhv

- $$\bar{u}\cdot\bar{v} = pr_N\bar{u}\cdot\bar{v}$$

$$\underline{|\bar{u}|\cdot\cos\alpha}\cdot|\bar{v}| = |pr_N\bar{u}|\cdot|\bar{v}|\cdot 1 =$$

$$= pr_N\bar{u}\cdot\bar{v}$$

$$(2 \cdot \bar{u}) \cdot \bar{v} = 2 \cdot (\bar{u} \cdot \bar{v})$$



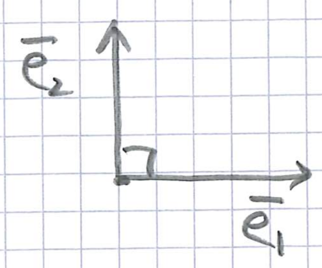
$$V.L. = |2 \cdot \bar{u}| \cdot |\bar{v}| \cos\alpha = 2\,|\bar{u}| \cdot |\bar{v}| \cdot \cos\alpha$$

$$= 2 \cdot (\bar{u} \cdot \bar{v}) = H.L.$$

<u>Sats</u> Om $\bar{u} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\bar{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

i en ON bas $\bar{e}_1, \bar{e}_2$ Så är

$$\boxed{\bar{u} \cdot \bar{v} = x_1 \cdot x_2 + y_1 \cdot y_2}$$



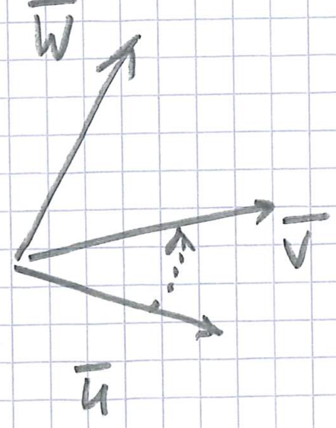
$$\bar{e}_1 \cdot \bar{e}_1 = |\bar{e}_1| \cdot |\bar{e}_1| = 1$$

$$\bar{e}_1 \cdot \bar{e}_2 = |\bar{e}_1| \cdot |\bar{e}_2| \cdot 0 = 0$$

$$\bar{e}_2 \cdot \bar{e}_2 = 1$$

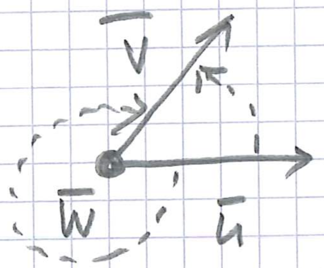
<u>Obs</u> $|\bar{u}| = \sqrt{\bar{u} \cdot \bar{u}} = \sqrt{x_1^2 + y_1^2}$

Ett högersystem



$(\bar{u}, \bar{v}, \bar{w})$ Obs ordning!

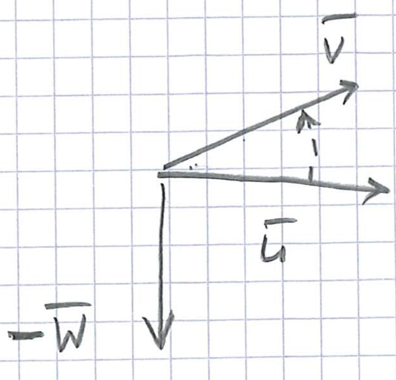
Syn från spetsen av $\bar{w}$ på planet där $\bar{u}, \bar{v}$ ligger:



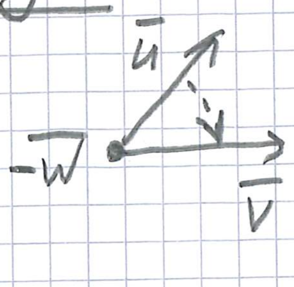
den minsta vridning som överför $\bar{u}$ till $\bar{v}$'s position ses moturs

Ett vänstersystem

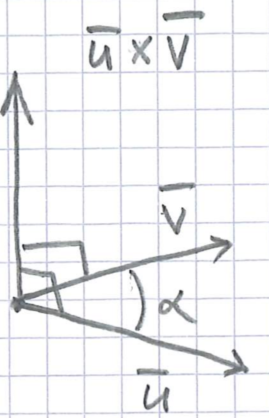
$(\bar{u}, \bar{v}, -\bar{w})$



Syn ... :



Vektorprodukt

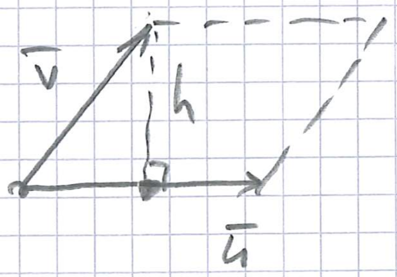


definieras av:

- $\bar{u} \times \bar{v} \perp \bar{u}, \bar{v}$
- $(\bar{u}, \bar{v}, \bar{u} \times \bar{v})$ är ett högersystem
- $|\bar{u} \times \bar{v}| = |\bar{u}| \cdot |\bar{v}| \cdot \sin\alpha$

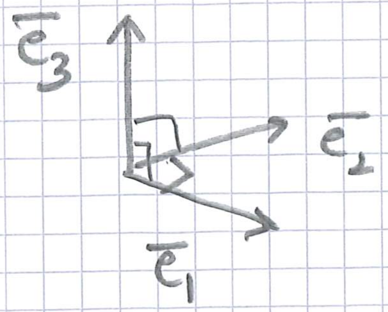
‖

<u>Arean</u> av



$h = |\bar{v}| \cdot \sin\alpha$

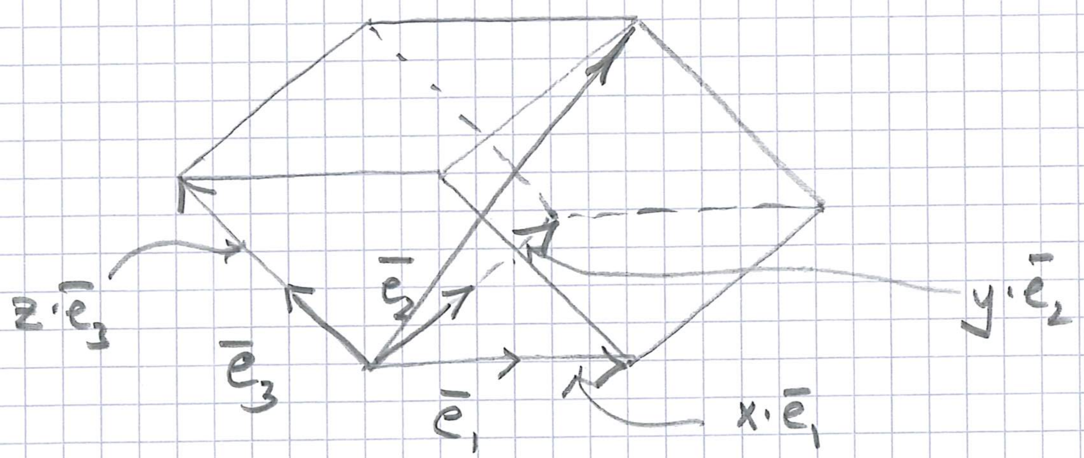
<u>Ex</u>: Anta att



- $(\bar{e}_1, \bar{e}_2, \bar{e}_3)$ är ett högersys.
- $|\bar{e}_i| = 1$ o $\bar{e}_i \perp \bar{e}_j$ för $i \neq j$

$$\Rightarrow \quad \bar{e}_1 \times \bar{e}_2 = \bar{e}_3$$
$$\bar{e}_1 \times \bar{e}_1 = \bar{0} \qquad \text{o s v}$$
$$\bar{e}_1 \times \bar{e}_3 = -\bar{e}_2$$

<u>En bas i rummet</u>:



$$\bar{w} = x \cdot \bar{e}_1 + y \cdot \bar{e}_2 + z \cdot \bar{e}_3$$

$$\bar{w} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{om } \bar{e}_1, \bar{e}_2, \bar{e}_3 \text{ är fixerad}$$

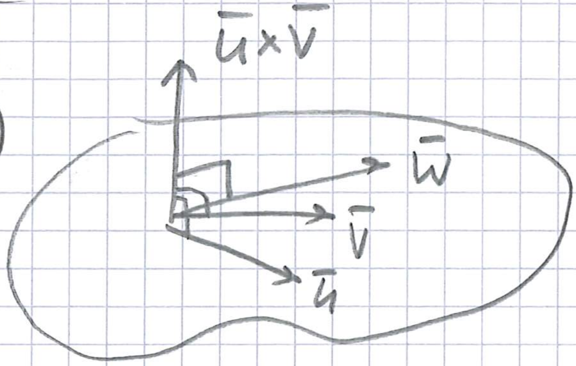
Trippelprodukt

$$(\bar{u}, \bar{v}, \bar{w}) = (\bar{u} \times \bar{v}) \cdot \bar{w} \quad (\text{def.})$$

$\bar{u}, \bar{v}, \bar{w}$ "ligger" i samma plan $\Leftrightarrow$

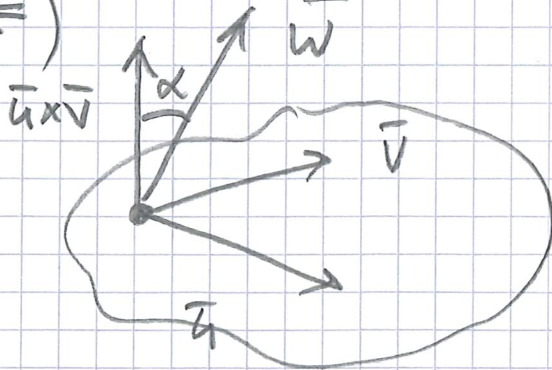
$$(\bar{u} \times \bar{v}) \cdot \bar{w} = 0$$

$(\Rightarrow)$



$$\Rightarrow (\bar{u} \times \bar{v}) \cdot \bar{w} = |\bar{u} \times \bar{v}| \cdot |\bar{w}| \cos \frac{\pi}{2} = 0$$

$(\Leftarrow)$



$$(\bar{u} \times \bar{v}) \cdot \bar{w} = \underset{\neq 0}{|\bar{u} \times \bar{v}|} \cdot \underset{\neq 0}{|\bar{w}|} \cdot \underset{\neq 0}{\cos \alpha}$$

$$\neq 0$$