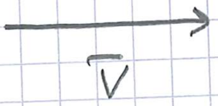
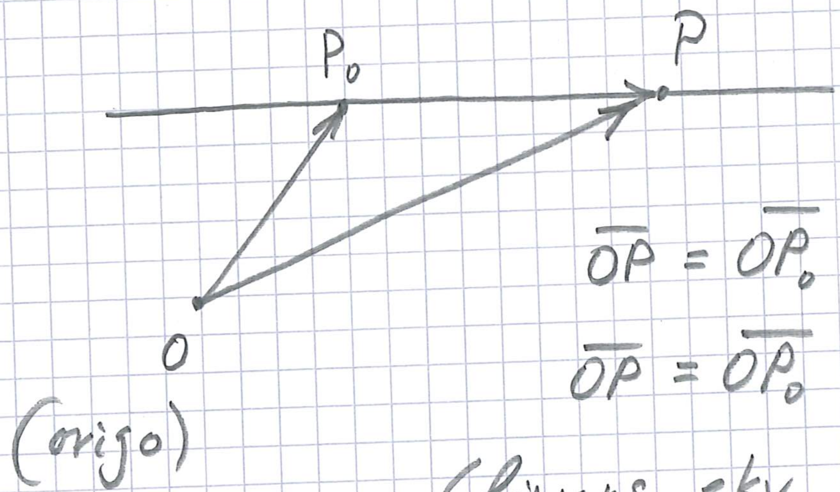


$\overline{v} \parallel L$

<u>Obs</u> $\forall P \in L$ $(\overline{P_0P} \parallel \overline{v})$ $\exists!\, t_p \in \mathbb{R}$ s.a

$$\overline{P_0P} = t_p \cdot \overline{v}$$

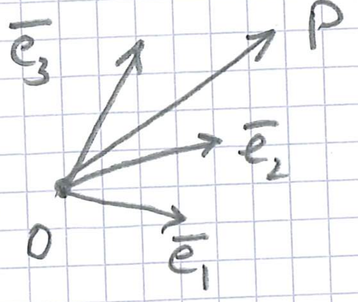


$\overline{OP} = \overline{OP_0} + \overline{P_0P}$ eller

$\overline{OP} = \overline{OP_0} + t \cdot \overline{v}$

(linjens ekv på <u>vektorform</u>)

Ett **Koordinatsystem** = origo O en bas

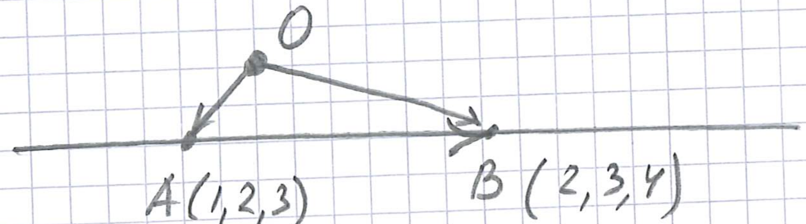


$$\left[\begin{array}{l} \overline{OP} = x\bar{e}_1 + y\bar{e}_2 + z\bar{e}_3 \\ P(x,y,z) \end{array}\right.$$

$$\underset{\overline{OP}\\P}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\overline{OP_0}\\P_0}{\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}} + \underset{t\cdot\bar{v}}{t\begin{pmatrix} a \\ b \\ c \end{pmatrix}} \quad \text{eller} \quad \begin{cases} x = x_0 + t a \\ y = y_0 + t b \\ z = z_0 + t c \end{cases}$$

$$\Rightarrow \quad \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Ex 1



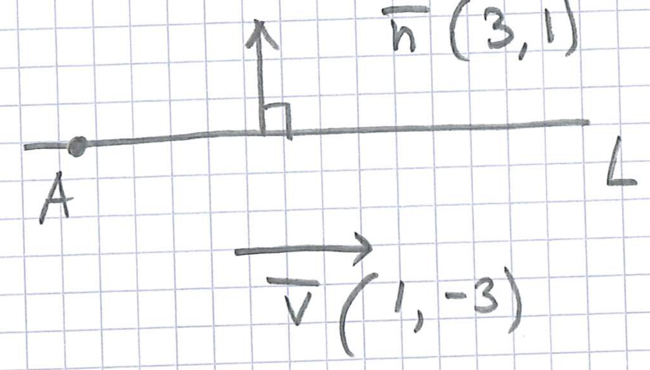
Obs $\overline{AB} = \overline{OB} - \overline{OA} = B - A = (1,1,1) \parallel L$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\cdot\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad t \in R$$

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$$

EX 2 (basen är en ON bas)

$3x + y - 2 = 0$



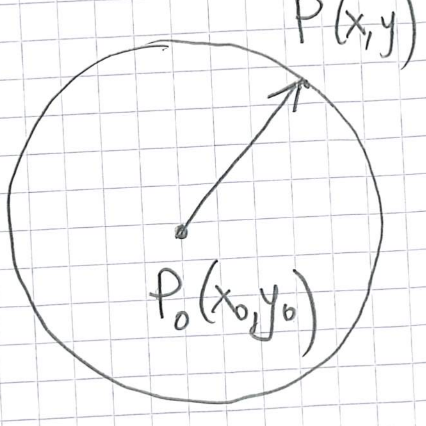
Obs om $x = 0$ så är $y = 2$.

Punkten $A(0,2) \in L$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ -3 \end{pmatrix} \text{ eller}$$

$$\begin{cases} x = t \\ y = 2 - 3t, \quad t \in R \end{cases}$$

1.



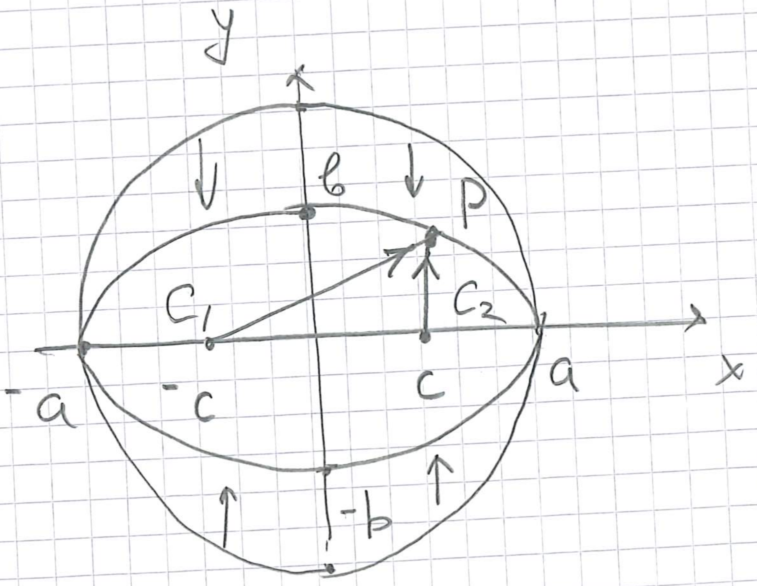
För alla P på kurvan gäller

$|\overline{P_0P}| = R$ (en konstant)

eller $(x-x_0)^2 + (y-y_0)^2 = R^2$

(en <u>cirkel</u> med <u>centrum</u> i P_0

<u>o</u> <u>radie</u> R)



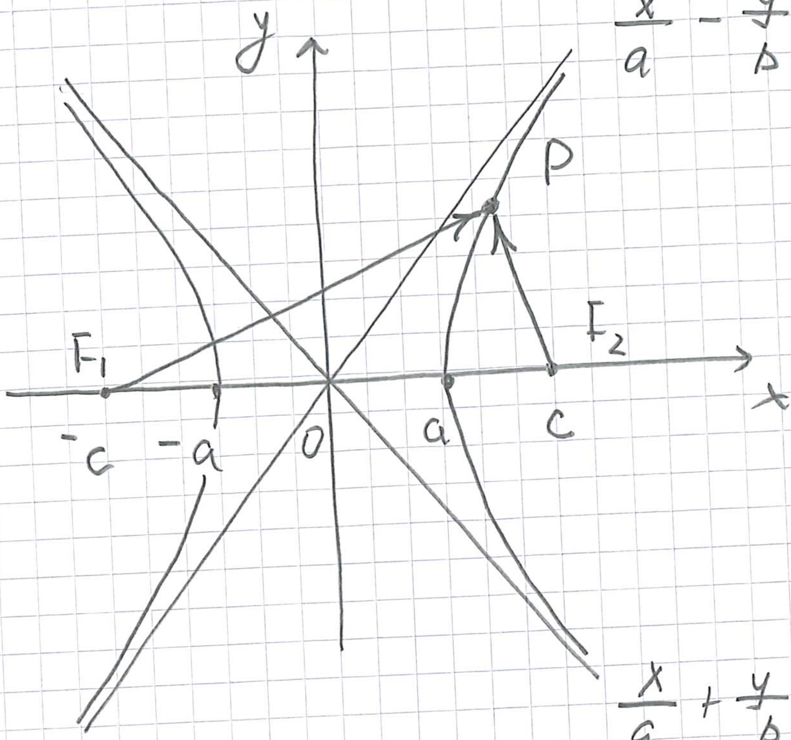
<u>Obs</u> $c = \sqrt{a^2 - b^2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$ (en <u>ellips</u>)

<u>Obs</u> $|\overline{c_1P}| + |\overline{c_2P}| = 2a$

för alla P på ellipsen.

3.



$\frac{x}{a} - \frac{y}{b} = 0$ (asymptot)

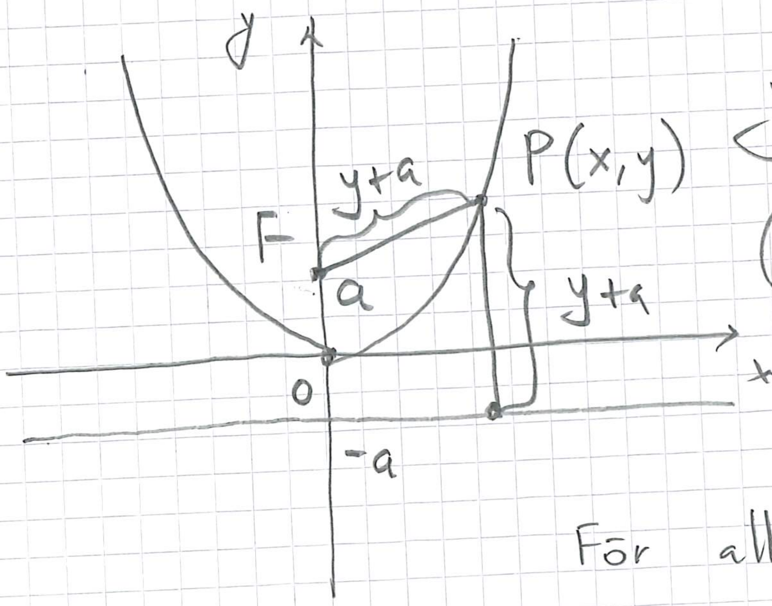
Obs $c = \sqrt{a^2+b^2}$

$\frac{x}{a} + \frac{y}{b} = 0$ (asymptot)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (en hyperbol)

Obs $\left| |\overline{F_1P}| - |\overline{F_2P}| \right| = 2a$

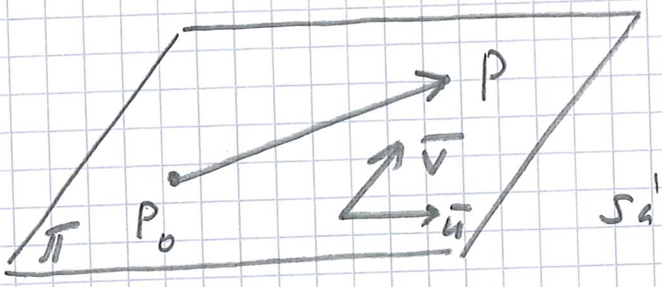
4.



$y = k \cdot x^2 \quad (k>0)$

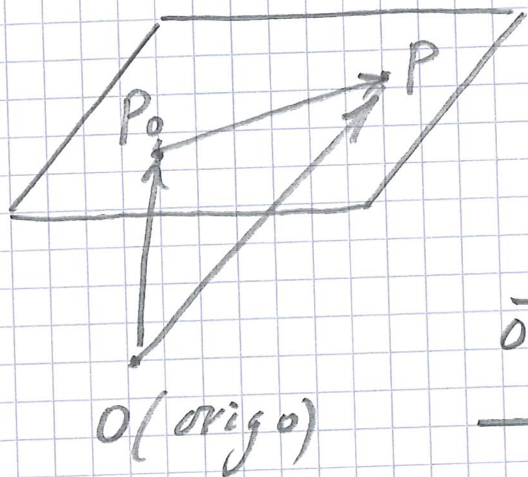
(en parabel)

För alla P på hyperbol



$\bar{u}, \bar{v}$ en bas i π

så är $\overline{P_0P} = t_p \cdot \bar{u} + s_p \cdot \bar{v}$

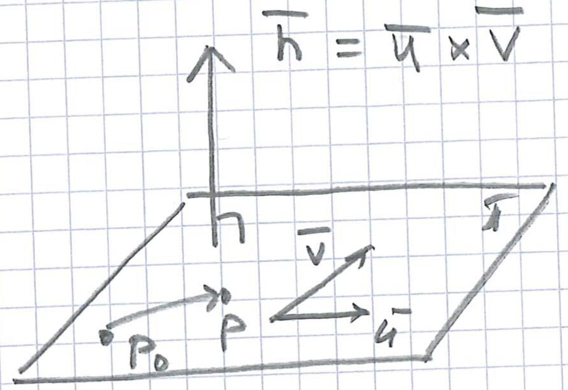


$\overline{OP} = \overline{OP_0} + \overline{P_0P}$

eller

$\overline{OP} = \overline{OP_0} + t_p \cdot \bar{u} + s_p \cdot \bar{v}$

$$\underset{\substack{\overline{OP} \\ P}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\substack{\overline{OP_0} \\ P_0}}{\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}} + t \underset{\bar{u}}{\begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}} + s \underset{\bar{v}}{\begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}}, \quad t, s \in R$$



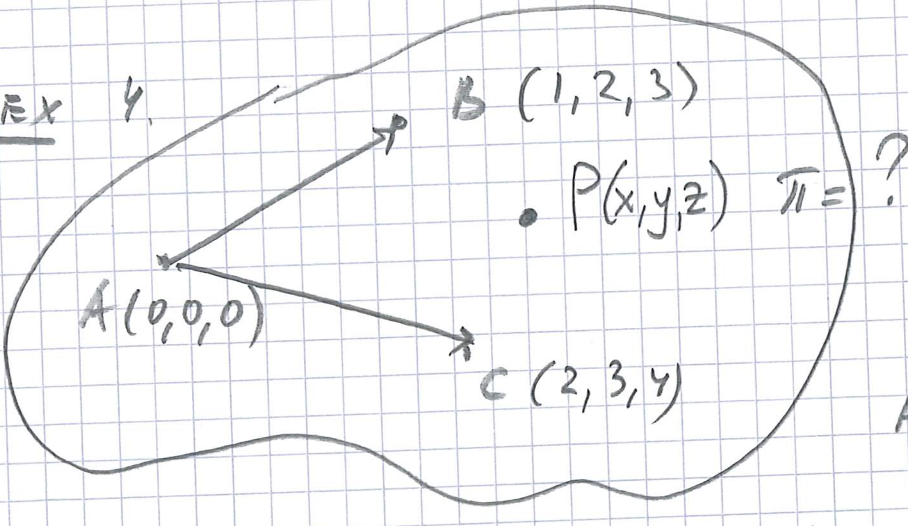
<u>Obs</u> (?) $\overline{P_0P} \perp \bar{n} \iff \overline{P_0P} \cdot \bar{n} = 0$

Om koordinatsystemet är <u>ortonormerat</u>

så är $Ax + By + Cz + D = 0$ planets ekv,

där $\bar{n}(A, B, C)$.

<u>Ex</u> 4.



$\overline{AB} = (1,2,3)$

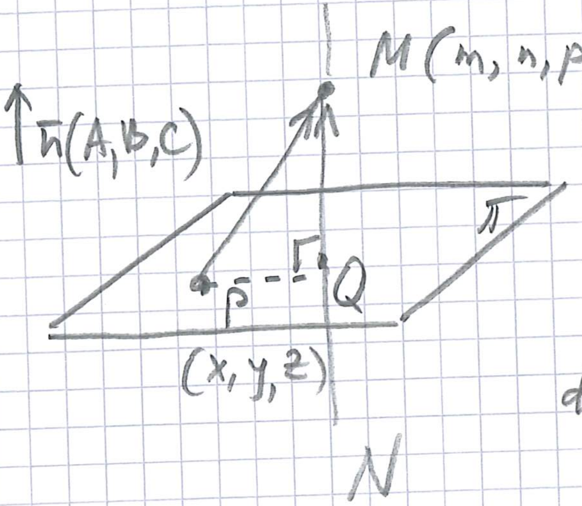
$\overline{AC} = (2,3,4)$

$$\bar{n} = \overline{AB} \times \overline{AC} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \perp \pi$$

$$\Rightarrow \pi: (-1)\cdot(x-0) + 2\cdot(y-0) + (-1)(z-0) = 0$$

eller $\quad -x + 2y - z = 0 \iff x - 2y + z = 0$

Ex 6 Avstånd d mellan en punkt o ett plan i rummet.



$$d = |\overline{QM}|$$

π: $Ax + By + Cz + D = 0$

$$d = \frac{|A\cdot m + B\cdot n + C\cdot p + D|}{\sqrt{A^2+B^2+C^2}}$$

(svar)

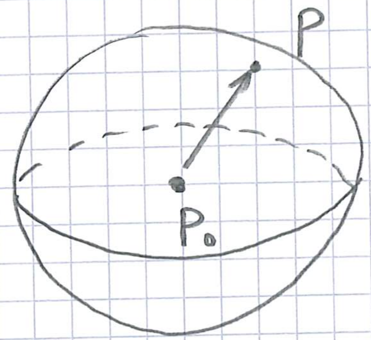
Obs $d = |\overline{QM}| = \left|\text{pr}_N \overline{PM}\right| =$

$$= \left|\frac{\overline{PM}\cdot\overline{n}}{|\overline{n}|^2}\cdot\overline{n}\right| = \frac{|\overline{PM}\cdot\overline{n}|}{|\overline{n}|} =$$

$$= \frac{|(m-x,\, n-y,\, p-z)\cdot(A, B, C)|}{\sqrt{A^2+B^2+C^2}} =$$

$$= \frac{|A(m-x) + B(n-y) + C(p-z)|}{\sqrt{A^2+B^2+C^2}} =$$

$$= \frac{|Am + Bn + Cp \overbrace{- Ax - By - Cz}^{= D}|}{\sqrt{A^2+B^2+C^2}}$$

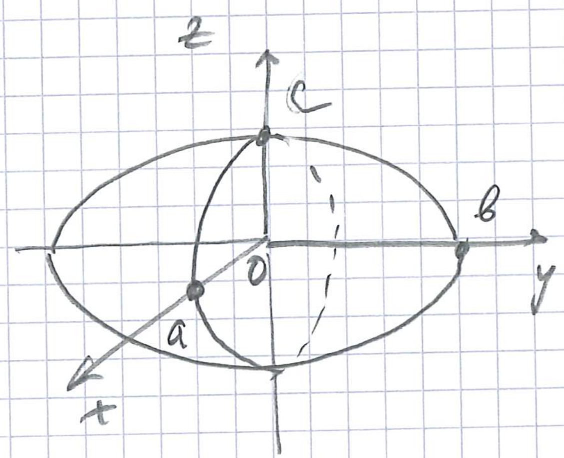


$$|\overline{P_0P}| = R \quad \forall P$$

eller

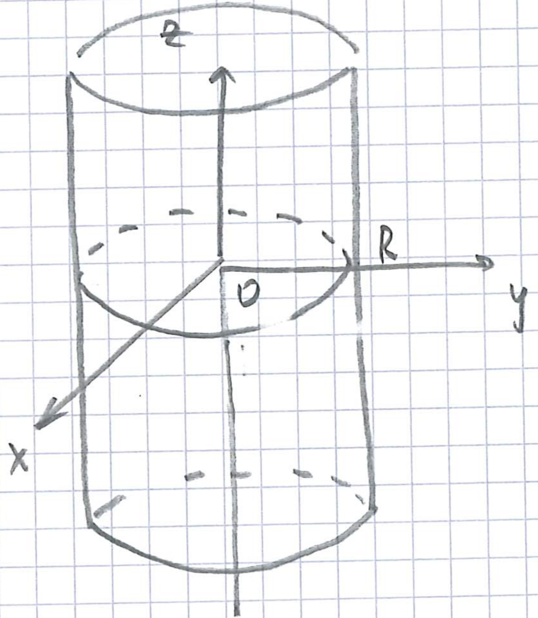
$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$$

(en <u>sfær</u>)



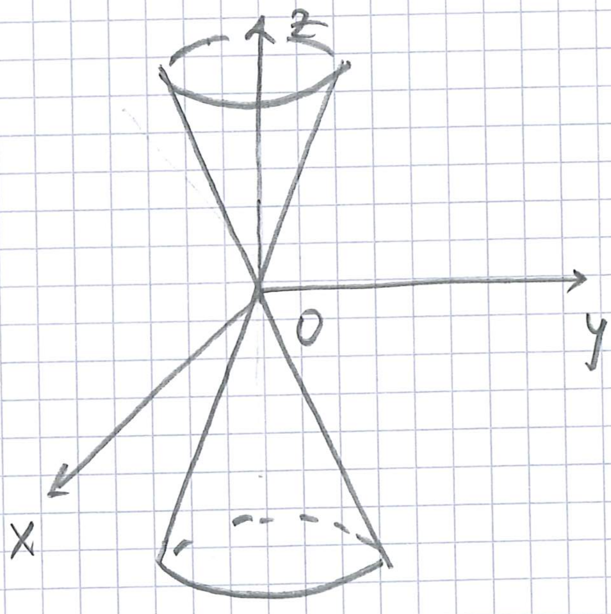
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

(en <u>ellipsoid</u>)



en <u>cylinder</u>

$$x^2+y^2=R^2$$



en <u>kon</u>

$$x^2+y^2=z^2$$