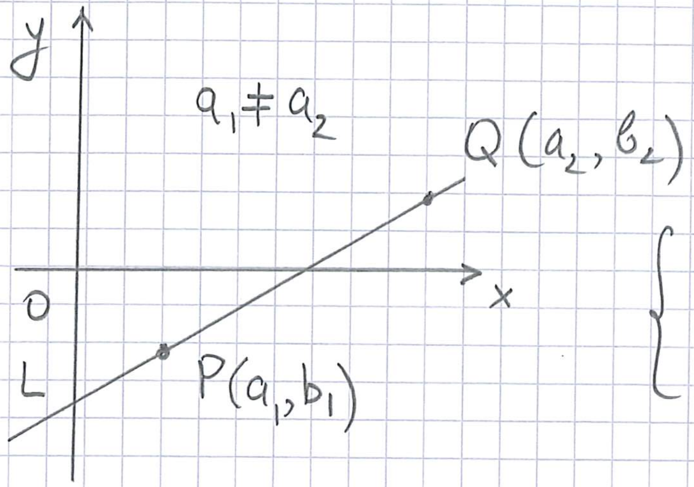
Inledande exempel:



$L: \; y = k \cdot x + m$

$$\begin{cases} b_1 = k \cdot a_1 + m & (1) \\ b_2 = k \cdot a_2 + m & (2) \end{cases}$$

$$(1)-(2): \quad b_1 - b_2 = k \cdot (a_1 - a_2) \Rightarrow k = \frac{b_1 - b_2}{a_1 - a_2} \curvearrowright (1)$$

$$\Rightarrow \quad m = b_1 - k \cdot a_1 = b_1 - \frac{b_1 - b_2}{a_1 - a_2} \cdot a_1 = \frac{a_1 b_2 - a_2 b_1}{a_1 - a_2}$$

Exempel 1

$$(1) \quad \begin{cases} x_1 + 2x_2 + x_3 = 0 & (1) \\ \quad\;\; x_2 + 3x_3 = -1 & (2) \\ \qquad\qquad\; x_3 = 1 & (3) \end{cases}$$

en trappa

$x_3 = 1 \curvearrowright (2)$:

$x_2 + 3 \cdot 1 = -1$

$\Rightarrow x_2 = -4 \curvearrowright (1)$:

bakåt substitution

$$x_1 + 2(-4) + 1 = 0 \Rightarrow x_1 = 8 - 1 = 7$$

Gausselimination

ett linjärt system	rad operationer $\longleftrightarrow$ ekvivalens	ett linjärt system på trappform

Exempel på elementära radoperationer:

(1) $\begin{cases} a_{11} x_1 + a_{12} x_2 = b_1 \\ a_{21} x_1 + a_{22} x_2 = b_2 \end{cases} \iff \begin{cases} a_{21} x_1 + a_{22} x_2 = b_2 \\ a_{11} x_1 + a_{12} x_2 = b_1 \end{cases}$

(2) $\begin{cases} a_{11} x_1 + a_{12} x_2 = b_1 \ (\times k) \\ a_{21} x_1 + a_{22} x_2 = b_2 \end{cases} \iff \begin{cases} k \cdot a_{11} x_1 + k \cdot a_{12} x_2 = k \cdot b_1 \\ a_{21} x_1 + a_{22} x_2 + b_2 \end{cases}$

$k \neq 0$

(3) $\begin{cases} a_{11} x_1 + a_{12} x_2 = b_1 \ (\times m) \\ a_{21} x_1 + a_{22} x_2 = b_2 \end{cases} \overset{+}{\iff} \begin{cases} a_{11} x_1 + a_{12} x_2 = b_1 \\ (a_{21} + m \cdot a_{11}) x_1 + (a_{22} + m a_{12}) x_2 \\ = b_2 + m \cdot b_1 \end{cases}$

Allmänt om succesiv elimination

Inhomogena system:

en generalisering av Exempel 1.

Eventuell omnumrering av obekanta:

Exempel:

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ \qquad\qquad\quad x_3 - 6x_4 = 7 \end{cases} \qquad (*)$$

Numrera om: $x_1' = x_1,\ x_2' = x_3,\ x_3' = x_2,\ x_4' = x_4$

$$(*) \quad \begin{cases} x_1' + 3x_2' + 2x_3' + 4x_4' = 5 \\ \qquad\quad x_2' \qquad\qquad - 6x_4' = 7 \end{cases}$$

Utökad matris:

$$\begin{array}{cccc} x_1' & x_2' & x_3' & x_4' \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & 4 & 5 \\ 0 & 1 & 0 & -6 & 7 \end{array}\right]$$

Homogena system:

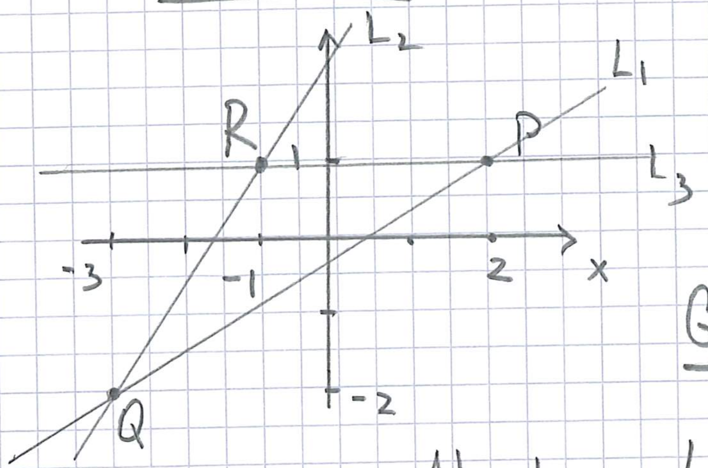
$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ \qquad\quad x_2 + 5x_3 + 6x_4 = 0 \end{cases}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & \underbrace{5 \quad 6}_{\text{2 kolonner}} & & 0 \end{array}\right]$$

Välj $x_3 = t,\ x_4 = s$ (två parametrar)

då är $x_2 = -5t - 6s$, $x_1 = -2(-5t-6s) - 3t - 4s =$
$= 7t + 8s$

Exempel 5



Finns det en rät linje som går igenom P, Q R?

Geometri $\Rightarrow$ nej

Algebra: $L: y = k \cdot x + b$

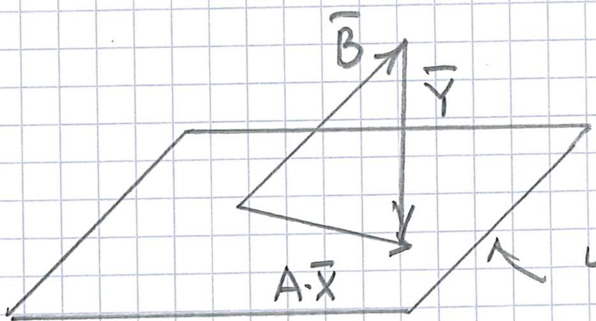
$$\begin{cases} 2k + b = 1 \\ -3k + b = -2 \\ -k + b = 1 \end{cases} \quad \emptyset$$

$\Rightarrow$ nej

Överbestämda ekvssystem o minsta kvadrat metod:

$A \cdot \bar{X} = \bar{B}$ saknar lösningar $\Rightarrow A\bar{X} \neq \bar{B} \quad \forall \bar{X}$

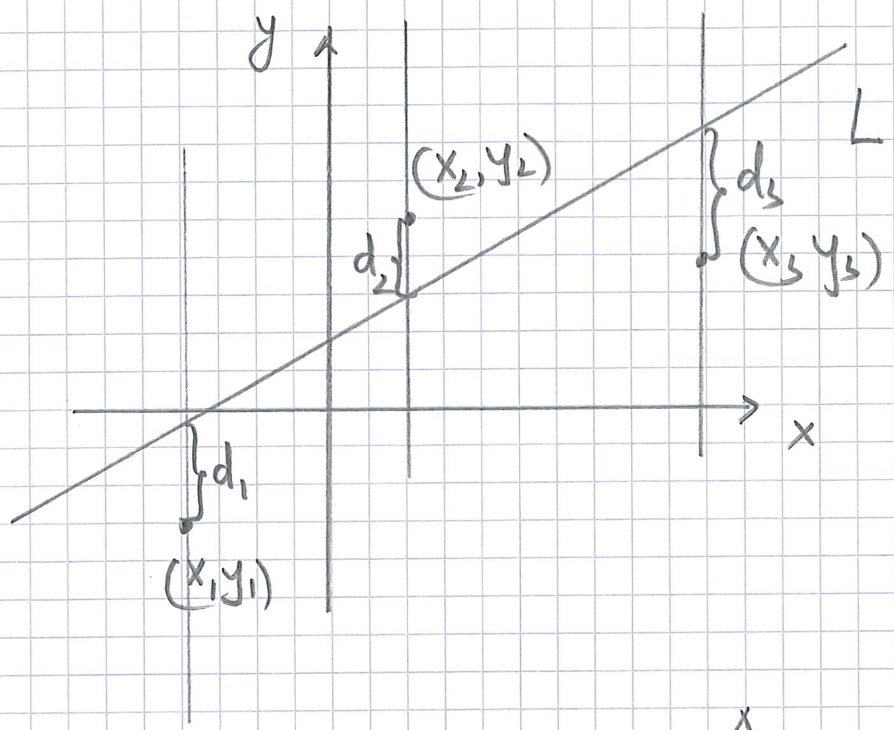
eller $A\bar{X} - \bar{B} = \bar{Y} \neq \bar{0} \quad \forall \bar{X}$



underrum $\{A \cdot \bar{X} \mid \bar{X}\} = \pi$

Det går att visa att det finns $\bar{Y}$ med minsta längd (tänk på 3-dimensionell bild)

$\bar{Y} \perp \pi$



$L: \; y = k \cdot x + b$

$$\begin{cases} k x_1 + b = y_1 \\ k x_2 + b = y_2 \\ k x_3 + b = y_3 \end{cases}$$

$$\overset{A}{\overset{\|}{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix}}} \cdot \overset{\overline{X}}{\overset{\|}{\begin{bmatrix} k \\ b \end{bmatrix}}} = \overset{\overline{B}}{\overset{\|}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}}$$

$$\overline{Y} = A \cdot \overline{X} - \overline{B} \qquad \underline{0}$$

$$|\overline{Y}|^2 = \underbrace{(k x_1 + b - y_1)^2}_{= d_1^2} + \underbrace{(k x_2 + b - y_2)^2}_{= d_2^2} + \underbrace{(k x_3 + b - y_3)^2}_{= d_3^2}$$

$\longrightarrow$ <u>min</u> (minimisera!)