Determinanter

Exempel 4:

(a) $\det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

(b) Sarrus regel (bara för (3x3)-matriser)

$$\left[\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right]$$

$$\Rightarrow \det A = a_{11}\, a_{22}\, a_{33} + a_{12}\, a_{23}\, a_{31} + a_{13}\, a_{21}\, a_{32}$$
$$- a_{31} \cdot a_{22} \cdot a_{13} - a_{32}\, a_{23}\, a_{11} - a_{33}\, a_{21}\, a_{12}$$

Eller

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}$$

+

Plus delen:

$a_{11}\,a_{22}\,a_{33} + a_{12}\,a_{23}\,a_{31} +$

$a_{13}\cdot a_{21}\cdot a_{32}$

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}$$

−

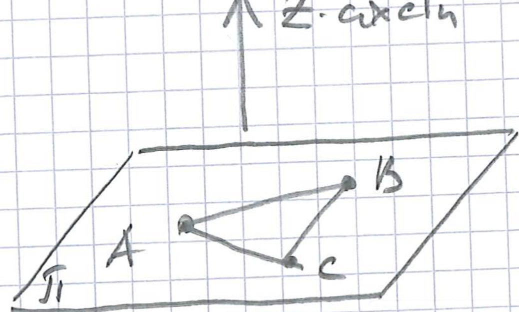
Minus delen:

$-a_{31}\,a_{22}\,a_{13} - a_{32}\cdot a_{23}\cdot a_{11}$

$-a_{33}\,a_{21}\,a_{12}$

$\det A = \underbrace{\text{Plus delen}} + \underbrace{\text{Minus delen}}$

Exempel 5:



$A(-1,2)$, $B(3,3)$, $C(2,-1)$

Finn arean av $\triangle ABC$.

Låt π vara x,y-planet i x,y,z-rummet d v s inför z-axeln.

Så är $\overline{AB} = (4,1,0)$, $\overline{AC} = (3,-3,0)$ i rummet.

Arean av $\triangle ABC = \frac{1}{2}\left|\overline{AB}\times\overline{AC}\right| =$

$$\frac{1}{2}\left|\begin{pmatrix}4\\1\\0\end{pmatrix}\times\begin{pmatrix}3\\-3\\0\end{pmatrix}\right| = \frac{1}{2}\left|(0,0,-15)\right| = \frac{15}{2}$$

<u>Egenskaper hos determinanter:</u>

(1) $$\det(\lambda\cdot\bar{a},\bar{b}) = \begin{vmatrix}\lambda a_1 & b_1\\ \lambda a_2 & b_2\end{vmatrix} = \lambda a_1 b_2 - \lambda a_2 b_1 =$$

$$= \lambda(a_1 b_2 - a_2 b_1) = \lambda\cdot\det(\bar{a},\bar{b})$$

$$\det(\bar{a}+\bar{b},\bar{c}) = \begin{vmatrix}(a_1+b_1) & c_1\\ (a_2+b_2) & c_2\end{vmatrix} =$$

$$= (a_1+b_1)c_2 - (a_2+b_2)c_1 = (a_1c_2 - a_2c_1) +$$

$$+ (b_1c_2 - b_2c_1) = \det(\bar{a},\bar{c}) + \det(\bar{b},\bar{c})$$

(2) $$\det(\bar{a},\bar{a}) = \begin{vmatrix}a_1 & a_1\\ a_2 & a_2\end{vmatrix} = a_1a_2 - a_2a_1 = 0$$

(3) $$\det(\bar{e}_1,\bar{e}_2) = \begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} = 1 - 0 = 1$$

(a) $\det(\bar{a}, \bar{b}) = \det(\bar{a} + k\cdot\bar{b}, \bar{b})$

$$V.L = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

$$HL = \begin{vmatrix} (a_1 + kb_1) & b_1 \\ (a_2 + kb_2) & b_2 \end{vmatrix} =$$

$$= (a_1 + kb_1) b_2 - (a_2 + kb_2) b_1 = a_1 b_2 - a_2 b_1$$

(b) $\det(\bar{a}, \bar{b}) = -\det(\bar{b}, \bar{a})$

$$V.L. = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

$$HL = -\begin{vmatrix} b_1 & a_1 \\ b_2 & a_2 \end{vmatrix} = -(b_1 a_2 - b_2 a_1) =$$

$$= a_1 b_2 - b_1 a_2$$

Exempel 7: $\det\begin{pmatrix} 1 & 2 & 0 \\ 2 & 7 & 1 \\ 3 & 5 & 2 \end{pmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 3 & -1 & 2 \end{vmatrix} =$

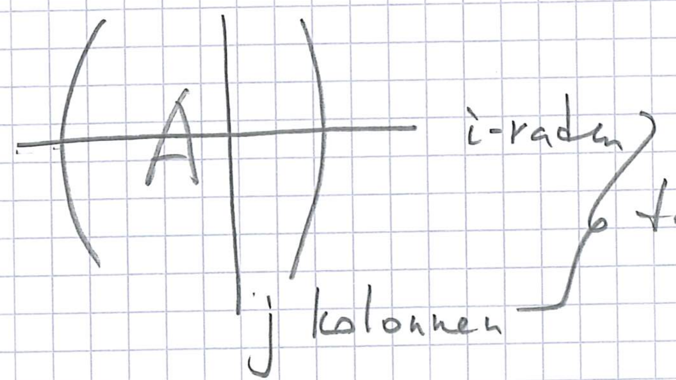
$\times(-2)$

Obs

$$- \begin{vmatrix} 1 & 0 & 0 \\ 3 & -1 & 2 \\ 2 & 3 & 1 \end{vmatrix} \underset{\times(-2)}{=} - \begin{vmatrix} 1 & 0 & 0 \\ -1 & -7 & 0 \\ 2 & 3 & 1 \end{vmatrix} = -(1\cdot(-7)\cdot 1) = \underline{7}$$

Utveckling efter en rad eller en kolonn

$\begin{pmatrix} & A & \end{pmatrix}$ är en $(n \times n)$ matris



$\Rightarrow A(i,j)$

Inför $D_{ij} = \det A(i,j)$

Algebraiska kompliment till platsen (i,j):

$$A_{ij} = (-1)^{i+j} \cdot D_{ij}$$

Sats:

$$\det A = a_{i1} \cdot A_{i1} + a_{i2} \cdot A_{i2} + \ldots + a_{in} \cdot A_{in}$$

(utveckling efter i:e raden)

$$\det A = a_{1j} \, A_{1j} + a_{2j} \cdot A_{2j} + \ldots + a_{nj} \cdot A_{nj}$$

(utveckling efter j:e kolonn)