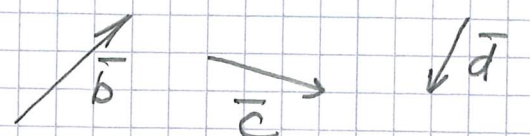
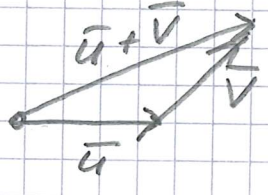
1.1 Låt



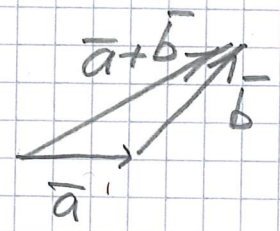
(a) Visa att $(\bar{a}+\bar{b})+\bar{c} = \bar{a}+(\bar{b}+\bar{c})$

Använd triangelregeln:

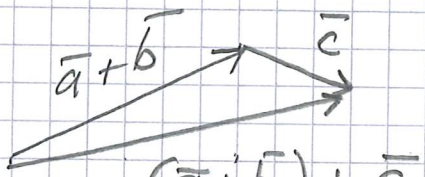


Börja med VL:

(i)

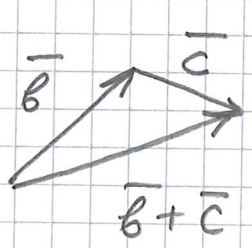


(ii)

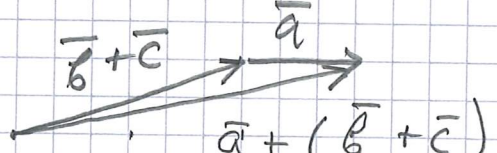


Fortsätt med HL:

(i)



(ii)



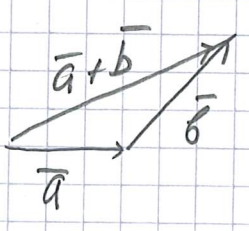
Obs att 1) $|VL| = |HL|$ (längderna är lika)

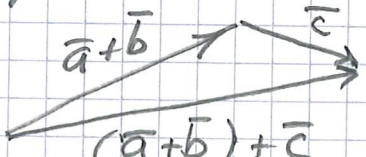
2) VL $\uparrow\downarrow$ HL har samma riktning $\Rightarrow$

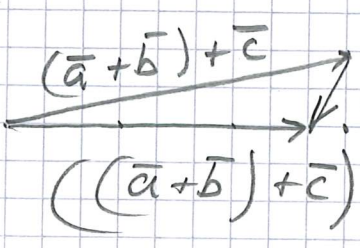
$VL = HL$.

(b) Bestäm $\bar{a}+\bar{b}+\bar{c}+\bar{d}$

Tillsätt parantes: $((\bar{a}+\bar{b})+\bar{c})+\bar{d}$ (till ex.)



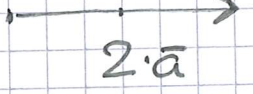




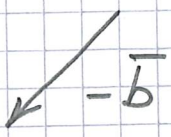
(c) Bestäm $2\bar{a} - \bar{b}$ grafiskt

Obs $2\bar{a} - \bar{b} = (2 \cdot \bar{a}) + (-\bar{b})$

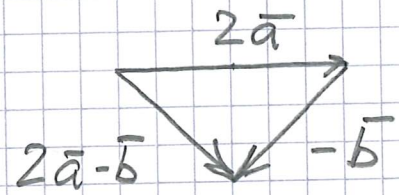
(i)



(ii)



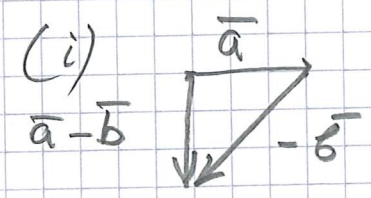
(iii)



(d) Bestäm $\bar{a} - \bar{b} - \bar{c}$ grafiskt.

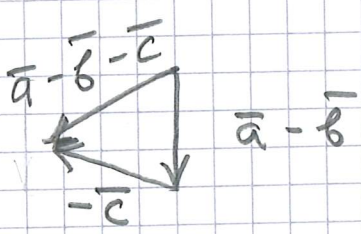
Obs $\bar{a} - \bar{b} - \bar{c} = (\bar{a} + (-\bar{b})) + (-\bar{c})$

(i)



(ii)

(iii)



1.3

Bestäm läget av en tredje punkt P s.a.

(a) $\overline{AP} = -\overline{BP}$

Betrakta den räta linje som går genom punkterna A och B.

Obs 1) P kan ligga bara på L ty $\overline{AP} \parallel \overline{BP}$

2) det finns tre möjligheter:

(i) till vänster av A:

passar ej ty $|\overline{AP}| < |-\overline{BP}|$

(ii) till höger av B:

passar ej ty $|\overline{AP}| > |-\overline{BP}|$

(iii) emellan A o B:

Välj P som mittpunkten av AB o notera att

$\overline{AP} = \overline{PB} = -\overline{BP}$

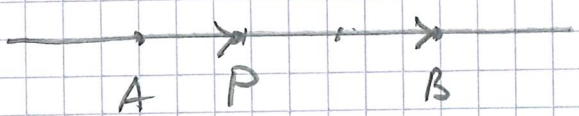
(b) $\overline{AP} = 4\overline{BP}$

Samma resonemang som i (a)

Välj P s.a. $|\overline{BP}| = \frac{1}{3} \cdot |\overline{AB}|$.

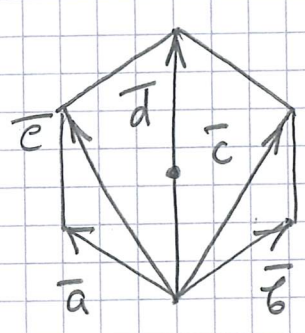
(c) $2\overline{AP} = -\overline{BP}$

Obs $-\overline{BP} = \overline{PB}$



Välj P emellan A o B s.a. $|\overline{AP}| = \frac{1}{3}|\overline{AB}|$

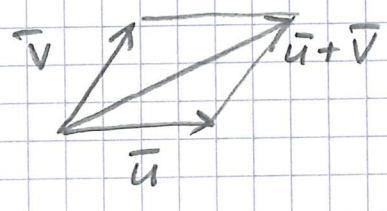
1.4



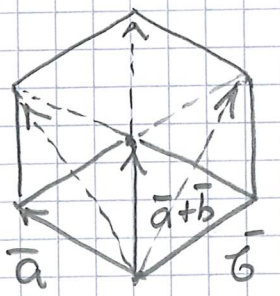
en regelbunden sexhörning

Uttryck $\bar{e}, \bar{c}, \bar{d}$ i $\bar{a}$ o $\bar{b}$.

Använd parallellogramregeln:



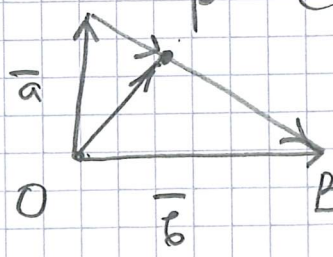
Obs 1) $2(\bar{a}+\bar{b}) = \bar{d}$ ty



2) $\bar{e} = (\bar{a}+\bar{b}) + \bar{a} = 2\bar{a} + \bar{b}$

3) $\bar{c} = (\bar{a}+\bar{b}) + \bar{b} = \bar{a} + 2\bar{b}$

1.7



(6) Bestäm P s.a. AP : PB = m : n

Obs 1) $\overline{OA} + \overline{AB} = \overline{OB} \Rightarrow$

$\overline{AB} = \overline{OB} - \overline{OA} = \bar{b} - \bar{a}$

2) $\overline{AP} = \frac{m}{m+n} \cdot \overline{AB}$ ty om $|\overline{AP}| = m \cdot x$ o $|\overline{PB}| = n \cdot x$

si är $|\overline{AB}| = mx + nx = (m+n)x$

o $\overline{AP}$ o $\overline{AB}$ har samma riktning.

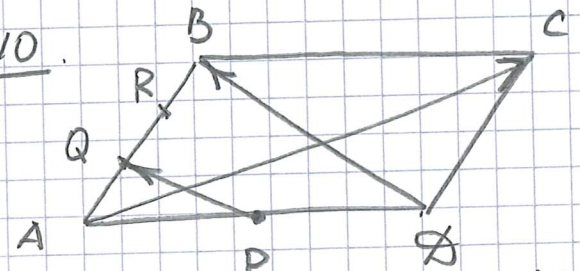
3) $\overline{OP} = \overline{OA} + \overline{AP} = \bar{a} + \frac{m}{m+n} \cdot (\bar{b} - \bar{a}) =$

$= \left(\bar{a} - \frac{m}{m+n}\bar{a}\right) + \frac{m}{m+n} \cdot \bar{b} = \left(1 - \frac{m}{m+n}\right) \cdot \bar{a} + \frac{m}{m+n} \cdot \bar{b} =$

$= \frac{n}{m+n}\bar{a} + \frac{m}{m+n}\bar{b}$

(a) om $m:n = 2:3$ si är $\overline{OP} = \frac{3}{5}\bar{a} + \frac{2}{5}\bar{b}$

1.10.



$AP = PD$

$AQ = QR = RB$

Uttryck $\overline{PQ}$ i $\overline{AC}$ o $\overline{DB}$:

Obs 1) $\overline{PQ} = \overline{PA} + \overline{AQ}$ (triangelregeln) (#)

2) $\overline{PA} = \frac{1}{2}\overline{DA}$ o $\overline{AQ} = \frac{1}{3}\overline{AB}$

Uttryck $\overline{DA}$ o $\overline{AB}$ i $\overline{AC}$ o $\overline{DB}$:

Obs 1) $\begin{cases} \overline{DB} = \overline{DA} + \overline{AB} \\ \overline{AC} = \overline{AD} + \overline{DC} = -\overline{DA} + \overline{AB} \end{cases}$ (*)

2) lös ut $\overline{DA}$ o $\overline{AB}$ ur (*) =>

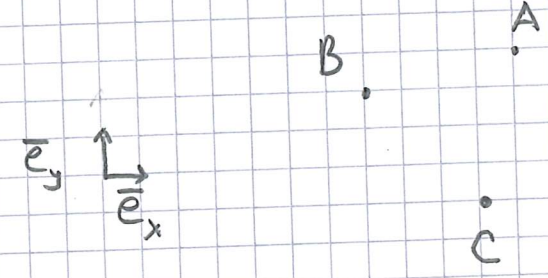
$\overline{AB} = \frac{1}{2}(\overline{DB} + \overline{AC})$ o $\overline{DA} = \frac{1}{2}(\overline{DB} - \overline{AC})$

3) Insättning i (#) ger

$$\overline{PQ} = \frac{1}{2}\overline{DA} + \frac{1}{3}\overline{AB} = \frac{1}{4}\left(\overline{DB} - \overline{AC}\right) + \frac{1}{6}\left(\overline{DB} + \overline{AC}\right)$$

$$= \frac{5}{12}\overline{DB} - \frac{1}{12}\overline{AB}$$

1.18



1) Rita som i 1.1. ☺

2) Bestäm koordinater: $\overline{AB} = -4\bar{e}_x - \bar{e}_y \quad (-4, -1)$

$$\overline{AB} + \overline{AC} = (-4\bar{e}_x - \bar{e}_y) + (-\bar{e}_x - 4\bar{e}_y) =$$

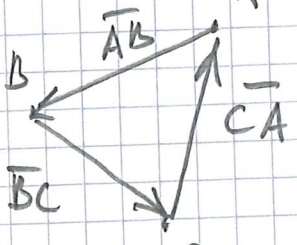
$$= -5\bar{e}_x - 5\bar{e}_y \quad (-5, -5)$$

$$\overline{AC} - \overline{BC} = (-\bar{e}_x - 4\bar{e}_y) - (3\bar{e}_x - 3\bar{e}_y) = -4\bar{e}_x - \bar{e}_y \quad (-4, -1)$$

$$2\overline{AB} = 2(-4\bar{e}_x - \bar{e}_y) = -8\bar{e}_x - 2\bar{e}_y \quad (-8, -2)$$

$$\overline{AB} + \overline{BC} + \overline{CA} = (-4\bar{e}_x - \bar{e}_y) + (3\bar{e}_x - 3\bar{e}_y) + (\bar{e}_x + 4\bar{e}_y)$$

$$= \bar{0} \quad (0, 0)$$

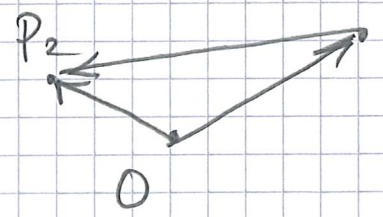


1.19 $\bar{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ☺ $\bar{v} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\Rightarrow \bar{u} + \bar{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$$2\bar{u} = 2 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

1.20 Bestäm koordinaterna av $\overline{P_1P_2}$

då $P_1 = (4, 3)$ o $P_2 = (-3, 1)$



Obs 1) $\overline{OP_1} = (4, 3)$

$\overline{OP_2} = (-3, 1)$

2) $\overline{P_1P_2} = \overline{OP_2} - \overline{OP_1} = (P_2) - (P_1) =$

$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \end{pmatrix}$.