4.5 Anta att $\det(\bar{a}, \bar{b}, \bar{c}) = d \neq 0$

(a) Beräkna $\det(3\bar{b} - 2\bar{a}, \bar{b}, \bar{c}) =$

$= \det(3\bar{b}, \bar{b}, \bar{c}) + \det(-2\bar{a}, \bar{b}, \bar{c}) =$

$= 3\det(\underbrace{\bar{b}, \bar{b}}_{=}, \bar{c}) - 2\det(\bar{a}, \bar{b}, \bar{c}) = 0 - 2d = -2d.$

(b) Beräkna $\det(\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}) =$

$\det(\bar{a}, \bar{b}+\bar{c}, \bar{c}) + \det(\bar{b}, \bar{b}+\bar{c}, \bar{c}) =$

$\det(\bar{a}, \bar{b}, \bar{c}) + \det(\bar{a}, \underbrace{\bar{c}, \bar{c}}_{=}) + \det(\underbrace{\bar{b}, \bar{b}}_{=}, \bar{c}) +$

$\det(\bar{b}, \underbrace{\bar{c}, \bar{c}}_{=}) = d + 0 + 0 + 0 = d.$

(c) Beräkna $\det(\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}+a) =$

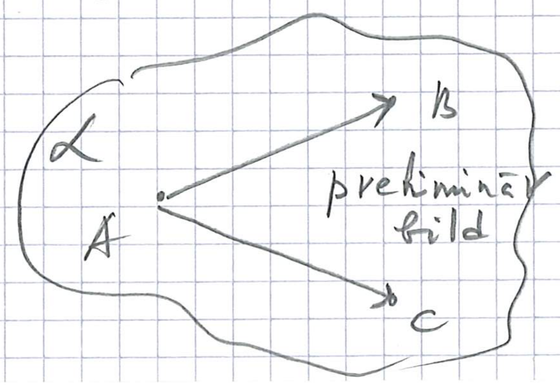
$= \det(\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}) + \det(\bar{a}+b, \bar{b}+\bar{c}, \bar{a}) =$

$= d + \det(\bar{a}, \bar{b}+\bar{c}, \bar{a}) + \det(\bar{b}, \bar{b}+\bar{c}, \bar{a}) =$

$= d + 0 + \det(\underbrace{\bar{b}, \bar{b}}_{=}, \bar{a}) + \det(\bar{b}, \bar{c}, \bar{a}) =$

$= d + 0 - \det(\bar{b}, \bar{a}, \bar{c}) = d + d = 2d.$

4.7 Undersök om punkterna $\overset{A}{(2,-1,1)}$, $\overset{B}{(1,0,2)}$, $\overset{C}{(3,-2,0)}$ o $\overset{D}{(-1,-3,1)}$ ligger i samma plan



Obs A, B, C ligger i samma plan α

$\bar{n} = \bar{AB} \times \bar{AC} \perp \alpha$ (om $\bar{n} \neq \bar{0}$)

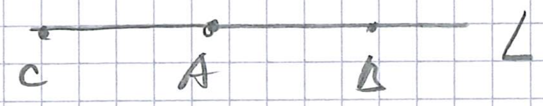
A, B, C, D ligger i samma plan $\Leftrightarrow$

$\bar{n} \cdot \overline{AD} = 0$

Räkna $\quad \overline{AB} = B - A = (-1, 1, 1)$, $\overline{AC} = C - A = (1, -1, -1)$

<u>Obs</u> $\quad \overline{AB} = -\overline{AC} \Rightarrow \bar{n} = \bar{0}$

A, B, C ligger på en rät linje L



$\Rightarrow$ A, B, C, D ligger i samma plan.

<u>Triangulär matris:</u>

<u>4.11</u> (a)
$$\begin{vmatrix} 1 & -1 & 2 \\ 3 & 3 & -1 \\ 2 & 1 & 1 \end{vmatrix} \begin{matrix} \times(-3), (-2) \\ \\ \\ \end{matrix} = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 6 & -7 \\ 0 & 3 & -3 \end{vmatrix} \text{ byte } =$$

Obs

$$= -\begin{vmatrix} 1 & -1 & 2 \\ 0 & 3 & -3 \\ 0 & 6 & -7 \end{vmatrix} \begin{matrix} \\ \times(-2) \\ \\ \end{matrix} = -\begin{vmatrix} 1 & -1 & 2 \\ 0 & 3 & -3 \\ 0 & 0 & -1 \end{vmatrix} = -1 \cdot 3 \cdot (-1) = 3$$

<u>Obs</u>

(b)
$$\begin{vmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 4 & 2 \\ 2 & 4 & 3 & 1 \\ 2 & 5 & 6 & 4 \end{vmatrix} \begin{matrix} \times(-1), (-2) \\ \\ \\ \\ \end{matrix} = \begin{vmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -3 & -1 \\ 0 & 1 & 0 & 2 \end{vmatrix} \begin{matrix} \\ \times(-1) \\ \\ \\ \end{matrix} =$$

Obs

$$= \begin{vmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & -1 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -3 & -1 \end{vmatrix} \begin{matrix} \\ \\ \times(-3) \\ \\ \end{matrix} = -\begin{vmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -4 \end{vmatrix} =$$

$$= -1\cdot 1\cdot(-1)\cdot(-4) = -4$$

(c)

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 1 \\ 1 & 1 & 1 & 5 \end{vmatrix} \text{ byte } \overset{\text{Obs}}{=} - \begin{vmatrix} 1 & 1 & 1 & 5 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 1 \\ 2 & 1 & 1 & 1 \end{vmatrix} \begin{matrix} \times(-1), (-2) \\ \\ \\ \\ \end{matrix} =$$

$$= - \begin{vmatrix} 1 & 1 & 1 & 5 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 3 & -4 \\ 0 & -1 & -1 & -9 \end{vmatrix} \text{ byte } \overset{\text{Obs}}{=} (+) \begin{vmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -1 & -9 \\ 0 & 0 & 3 & -4 \\ 0 & 2 & 0 & -4 \end{vmatrix} \begin{matrix} \\ \times(2) \\ \\ \\ \end{matrix} =$$

$$\begin{vmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -1 & -9 \\ 0 & 0 & 3 & -4 \\ 0 & 0 & -2 & -22 \end{vmatrix} \begin{matrix} \\ \\ \\ \times 1 \end{matrix} = \begin{vmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -1 & -9 \\ 0 & 0 & 1 & -26 \\ 0 & 0 & -2 & -22 \end{vmatrix} \begin{matrix} \\ \\ \times 2 \\ \\ \end{matrix} =$$

$$= \begin{vmatrix} 1 & 1 & 1 & 5 \\ 0 & -1 & -1 & -9 \\ 0 & 0 & 1 & -26 \\ 0 & 0 & 0 & -74 \end{vmatrix} = 1\cdot(-1)\cdot 1\cdot(-74) = 74$$

(d)

$$\begin{vmatrix} 1 & 2 & 1 & 3 \\ -1 & 1 & 3 & 2 \\ 1 & 0 & 2 & 3 \\ -1 & 1 & 1 & 4 \end{vmatrix} \begin{matrix} \times 1, (-1) \\ \\ \\ \\ \end{matrix} = \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 3 & 4 & 5 \\ 0 & -2 & 1 & 0 \\ 0 & 3 & 2 & 7 \end{vmatrix} \begin{matrix} \\ \times(-1), 1 \\ \\ \\ \end{matrix} =$$

$$= \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 3 & 4 & 5 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & -2 & 2 \end{vmatrix} \text{byte} = - \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 5 & 5 \\ 0 & 3 & 4 & 5 \\ 0 & 0 & -2 & 2 \end{vmatrix} \begin{matrix} \\ \times(-3) \\ \\ \\ \end{matrix} =$$

$$= - \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & -11 & -10 \\ 0 & 0 & -2 & 2 \end{vmatrix} \begin{matrix} \\ \\ \\ \times(-5) \end{matrix} = - \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & -1 & -20 \\ 0 & 0 & -2 & 2 \end{vmatrix} \begin{matrix} \\ \\ \times(-2) \\ \\ \end{matrix} =$$

$$= - \begin{vmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 5 & 5 \\ 0 & 0 & -1 & -20 \\ 0 & 0 & 0 & 42 \end{vmatrix} = -1 \cdot 1 \cdot (-1) \cdot 42 = 42$$

4.14 Visa att om $a+b+c+d=0$ så är

$$D = \begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix} = 0$$

$$D = \begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix} \begin{matrix} \\ \times 1 \\ \times 1 \\ \times 1 \end{matrix} = \begin{vmatrix} \overset{0}{(a+b+c+d)} & \overset{0}{(a+b+c+d)} & \overset{0}{(a+b+c+d)} & \overset{0}{(a+b+c+d)} \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} \boxed{0 \quad 0 \quad 0 \quad 0} \\ b \quad c \quad d \quad a \\ c \quad d \quad a \quad b \\ d \quad a \quad b \quad c \end{vmatrix} = 0$$

4.19

(a)
$$\begin{vmatrix} 1 & 0 & 3 & -1 \\ 1 & 2 & 4 & -3 \\ 0 & 1 & 1 & 1 \\ 3 & -1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & 0 \\ 1 & 2 & 4 & -2 \\ 0 & 1 & 1 & 1 \\ 3 & -1 & 0 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & -2 \\ 0 & 1 & 1 & 1 \\ 3 & -1 & -9 & 5 \end{vmatrix} =$$

$= \Big|$ utveckla det efter 1:a raden $\Big|$

$$= 1 \cdot \begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & 1 \\ -1 & -9 & 5 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ -1 & -9 & 4 \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 2 \\ 17 & -9 & 4 \end{vmatrix} =$$

$= \Big|$ utveckla det efter 1:a raden $\Big| =$

$$= 1 \cdot (-1) \begin{vmatrix} -1 & 2 \\ 17 & 4 \end{vmatrix} = -(-4-34) = 38$$

(d)
$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 4 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 4 & 0 & 0 \end{vmatrix} = -2 \begin{vmatrix} 0 & 3 \\ 4 & 0 \end{vmatrix} = -2(-12) = 24$$

4.21

$$\begin{cases} u = 3x + 2y \\ v = x - 2y \end{cases}$$

avbildar triangeln med hörn i punkterna $A(0,0)$, $B(2,2)$ o $C(2,4)$ i x,y-planet på en triangel i u,v-planet. Bestäm arean av bilden.

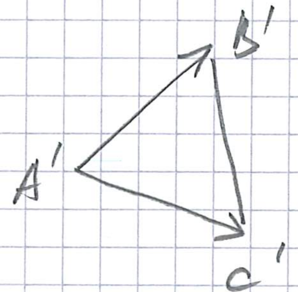
Finn bilder av punkterna A, B, C i u,v-planet.

$$\overset{A}{(0,0)} \to \overset{A'}{(0,0)}, \quad \overset{B}{(2,2)} \to (3\cdot 2+2\cdot 2,\ 2-2\cdot 2) = \overset{B'}{(10,-2)}$$

$$\overset{C}{(2,4)} \to (3\cdot 2+2\cdot 4,\ 2-2\cdot 4) = \overset{C'}{(14,-6)}$$

Triangeln $A'B'C'$ av bildtriangeln.

Räkna arean S' av $A'B'C'$:



$$S = \frac{1}{2}\left|\overline{A'B'} \times \overline{A'C'}\right|$$ (Sätt 3:e koordinaten lika med 0)

$$\overline{A'B'} = B' - A' = (10,-2)$$

$$\overline{A'C'} = C' - A' = (14,-6)$$

$$\overline{A'B'} \times \overline{A'C'} = \begin{pmatrix}10\\-2\\0\end{pmatrix} \times \begin{pmatrix}14\\-6\\0\end{pmatrix} = \begin{pmatrix}0\\0\\-32\end{pmatrix}$$

$$\Rightarrow S = \frac{1}{2}\cdot 32 = 16$$

4.25 A är en kvadratisk matris s.a.

$A^2 = A$. Vilka värden kan $\det A$ anta?

Repetera att $\det(A\cdot B) = \det A \cdot \det B$

$$\Rightarrow (\det A)^2 = \det A \quad \text{eller}$$

$$\det A \cdot (\det A - 1) = 0$$

Vi får två värden 0 eller 1