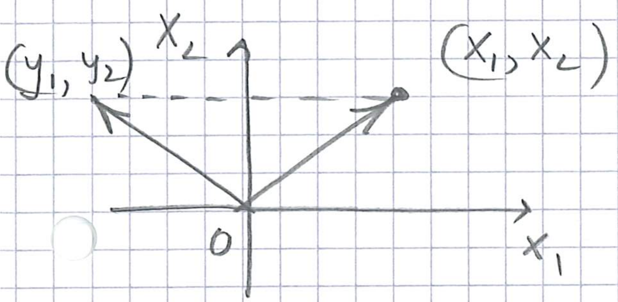
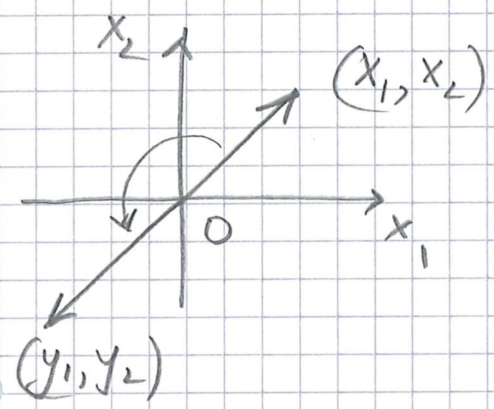
2.31 Beskriv geometriskt de linjära avbildningar som hör till matriserna. $\boxed{Y = M \cdot X}$

(a) $M_a = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 \\ x_2 \end{pmatrix}$



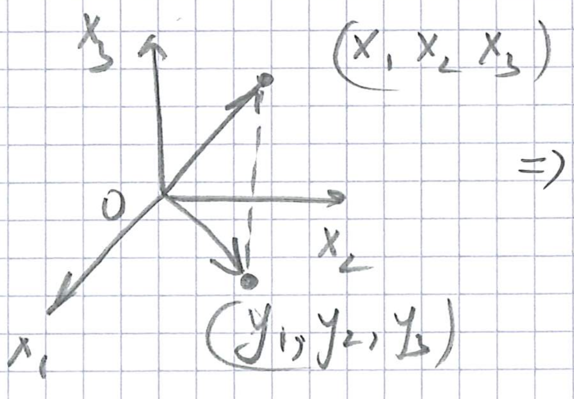
$\Rightarrow$ Spegling i x_2-axeln.

(b) $M_b = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 \\ -x_2 \end{pmatrix}$



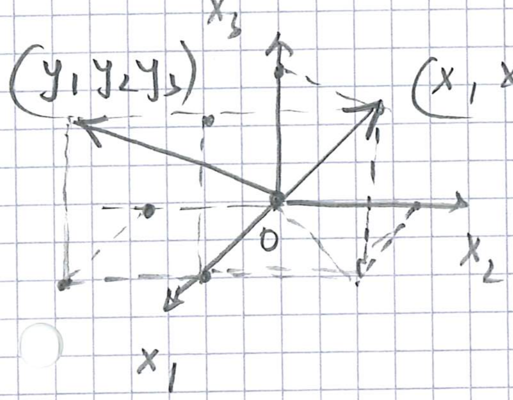
$\Rightarrow$ Vridning vinkeln π i origo.

(c) $M_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ 0 \end{pmatrix}$



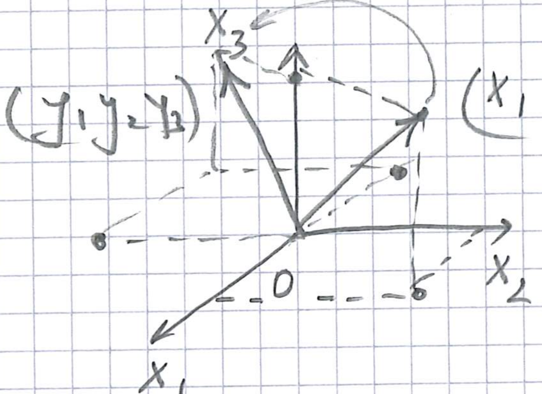
$\Rightarrow$ ortogonal projektion på x_1, x_2-planet.

(d) $M_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ -x_2 \\ x_3 \end{pmatrix}$



$\Rightarrow$ Spegling i $x_1 x_3$-planet.

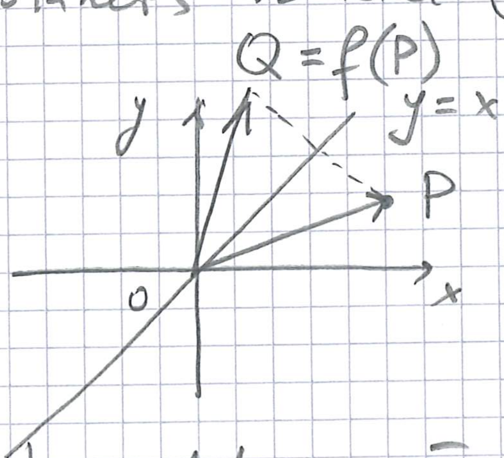
(e) $M_e = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_1 \\ -x_2 \\ x_3 \end{pmatrix}$



$\Rightarrow$ Vridning vinkeln π kring x_3-axeln.

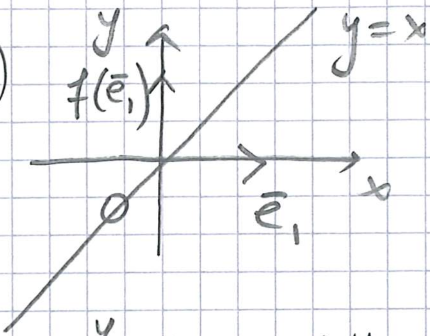
2.32. Bestäm matrisen för den lin. avbildning f som speglar planets vektorer (punkter) i linjen

(a) $L_a : y = x$



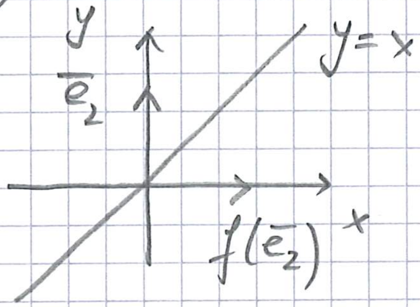
Kolla bilder av basvektorer $\bar{e}_1, \bar{e}_2$!

(i)



$\underline{\text{Obs}}$ $f(\bar{e}_1)=\begin{pmatrix}0\\1\end{pmatrix}$

(ii)



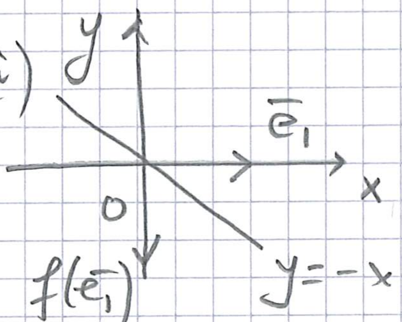
$\underline{\text{Obs}}$ $f(\bar{e}_2)=\begin{pmatrix}1\\0\end{pmatrix}$

Repetera att $M=\left[\, f(\bar{e}_1) \,\vdots\, f(\bar{e}_2) \,\right]$

$$\underline{=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}}$$

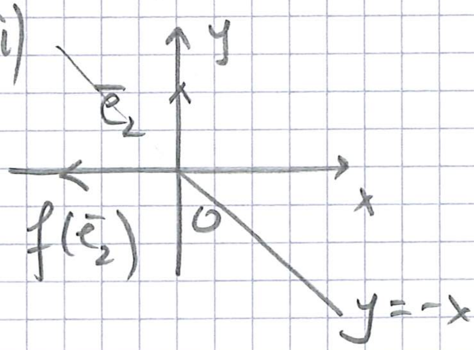
(b) $y=-x$. Handla som ovan.

(i)



$\Rightarrow f(\bar{e}_1)=\begin{pmatrix}0\\-1\end{pmatrix}$

(ii)



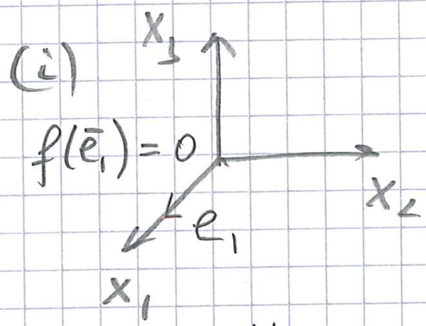
$\Rightarrow f(\bar{e}_2)=\begin{pmatrix}-1\\0\end{pmatrix}$

$\Rightarrow M=\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$

$\underline{2.35}$ Bestäm matrisen för den lin. avbildning f som är ortogonala projektionen på

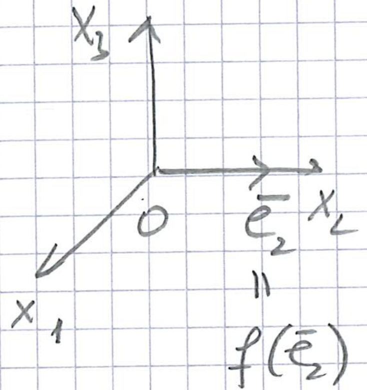
(a) planet $x_1 = 0$. Handla som i 2.32

(i)



$\Rightarrow$ Obs $f(\bar{e}_1) = \bar{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(ii)

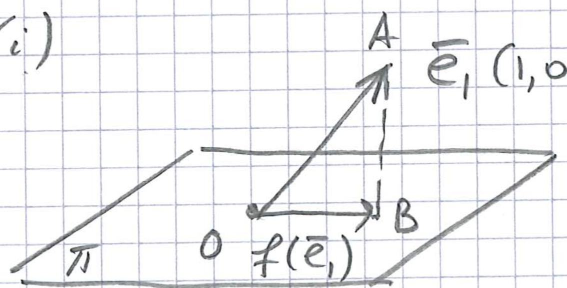


$\Rightarrow$ Obs $f(\bar{e}_2) = \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(iii) $f(\bar{e}_3) = \bar{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\Bigg| \Rightarrow M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) planet $2x_1 + 2x_2 + x_3 = 0$ (π)

(i)



$\bar{e}_1\,(1,0,0)$ Obs $\bar{n}\,(2,2,1) \perp \pi$ $\underline{o}$

$|\bar{n}| = 3$

$\overline{BA} = \text{proj}_{\bar{n}}\, \bar{e}_1 = \dfrac{\bar{n} \cdot \bar{e}_1}{|\bar{n}|^2} \cdot \bar{n}$

$= \dfrac{2}{9}(2,2,1),$

$f(\bar{e}_1) = \text{proj}_{\pi}\, \bar{e}_1 = \overline{OB} = \overline{OA} + \overline{AB} = (1,0,0) - \frac{2}{9}(2,2,1) =$

$= \left(\dfrac{5}{9}, \dfrac{-4}{9}, \dfrac{-2}{9}\right) = \dfrac{1}{9}\begin{bmatrix} 5 \\ -4 \\ -2 \end{bmatrix}$

Analogt för $\bar{e}_2$ o $\bar{e}_3$: Obs $\bar{n}\cdot\bar{e}_2 = 2$, $\bar{n}\cdot\bar{e}_3 = 1$

$$f(\bar{e}_2) = (0,1,0) - \frac{2}{9}(2,2,1) = \left(-\frac{4}{9}, \frac{5}{9}, -\frac{2}{9}\right) = \frac{1}{9}\begin{pmatrix}-4\\5\\-2\end{pmatrix}$$

$$f(\bar{e}_3) = (0,0,1) - \frac{1}{9}(2,2,1) = \left(-\frac{2}{9}, -\frac{2}{9}, \frac{8}{9}\right) =$$

$$= \frac{1}{9}\begin{pmatrix}-2\\-2\\8\end{pmatrix} \Rightarrow M = \frac{1}{9}\begin{bmatrix}5 & -4 & -2\\-4 & 5 & -2\\-2 & -2 & 8\end{bmatrix}$$

2.36 En linjär avbildning f i planet avbildar $\bar{e}_1$ på $f(\bar{e}_1) = \bar{e}_1 - \bar{e}_2$ o $\bar{e}_2$ på $f(\bar{e}_2) = e_1$. Bestäm avbildningens matris o bilden av $(2\ 3)^t$.

$\mathcal{E} = (\bar{e}_1, e_2)$ Så är $A = \left[[f(\bar{e}_1)]_{\mathcal{E}} \mid [f(e_2)]_{\mathcal{E}} \right]$

$$[f(\bar{e}_1)]_{\mathcal{E}} = \begin{pmatrix}1\\-1\end{pmatrix}, \quad [f(\bar{e}_2)]_{\mathcal{E}} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix}1 & 1\\-1 & 0\end{bmatrix}.$$

Bilden av $(2\ 3)^t$ är $A\cdot\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}1 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}5\\-2\end{bmatrix}$

2.37 Bestäm den vridningsmatris som överför vektorn $(3\ 4)^t$ på $(5, 0)^t$.

Obs $A = \begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix} \Rightarrow \begin{bmatrix}5\\0\end{bmatrix} = \begin{bmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{bmatrix}\cdot\begin{bmatrix}3\\4\end{bmatrix}$

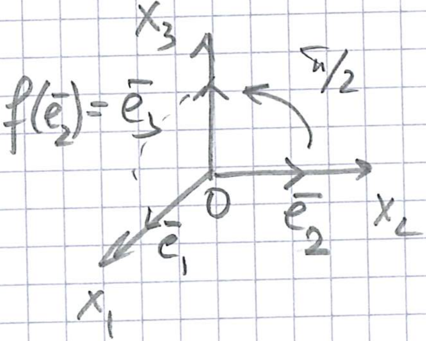
eller $\begin{cases} 3\cos\varphi - 4\sin\varphi = 5 & (1) \\ 3\sin\varphi + 4\cos\varphi = 0 & (2) \end{cases}$ lös ut $\cos\varphi, \sin\varphi$.

$(2) \Rightarrow \quad -\sin\varphi = \frac{4}{3}\cos\varphi \;\rightarrow\; (1): \; 3\cos\varphi + \frac{16}{3}\cos\varphi = 5$

eller $\cos\varphi = 3/5 \quad$ o $\quad \sin\varphi = -\frac{4}{3}\cdot\frac{3}{5} = -\frac{4}{5}$

$$\Rightarrow A = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$

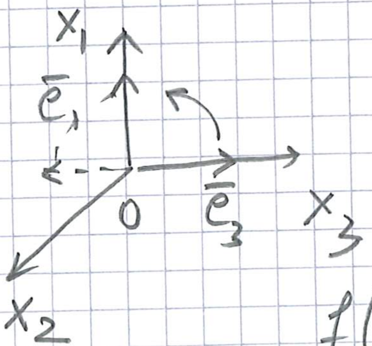
<u>2.38</u> A <u>vridningen</u> $\pi/2$ kring x_1-axeln så att $\bar{e}_2$ avbildas på $\bar{e}_3$



$\Rightarrow$ Ohs $\quad \begin{cases} f(\bar{e}_1) = \bar{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ f(\bar{e}_2) = \bar{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ f(\bar{e}_3) = -\bar{e}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \end{cases}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \Leftarrow$$

B <u>vridningen</u> $\pi/2$ kring x_2-axeln så att $\bar{e}_3$ avbildas på $\bar{e}_1$.

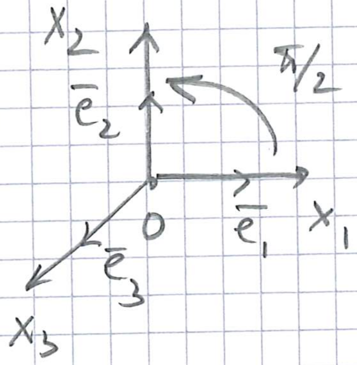


$\Rightarrow \quad f(\bar{e}_1) = -\bar{e}_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \quad f(\bar{e}_2) = \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$f(\bar{e}_3) = \bar{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

2.45

(a) A vridning $\pi/2$ kring X_3-axeln sett moturs från spetsen av X_3.



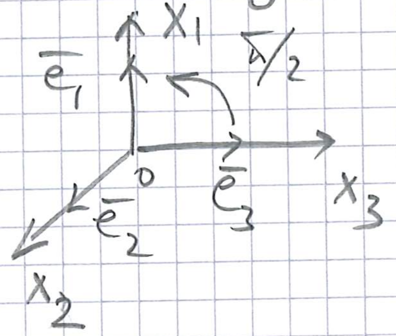
$$\Rightarrow \quad f(\bar{e}_1) = \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$f(\bar{e}_2) = -\bar{e}_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$f(\bar{e}_3) = \bar{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Leftarrow$$

B vridning $\pi/2$ kring X_2-axeln sett moturs från spetsen av X_2-axeln



$$\Rightarrow \quad f(\bar{e}_1) = -\bar{e}_3 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$f(\bar{e}_2) = \bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$f(\bar{e}_3) = \bar{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

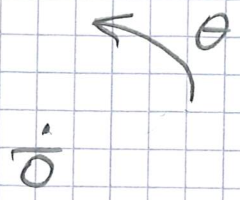
$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad \Leftarrow$$

(b) $$B \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(c) $$A \cdot B = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

2.49

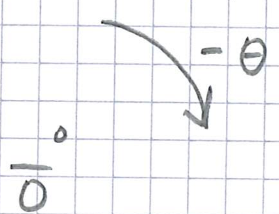
(a) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$



$A^2 = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$ vridning 2θ kring origo.

$A^n = \begin{bmatrix} \cos(n\theta) & -\sin(n\theta) \\ \sin(n\theta) & \cos(n\theta) \end{bmatrix}$ vridning $n \cdot \theta$ kring origo.

(b) $A^t = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$



vridning $-\theta$ kring origo
(eller θ medurs)

(c) $AA^t = I \Leftrightarrow A^t A = I$ är identitet. (enhetsmatris)

$AA^t \bar{x} = \bar{x} \Leftrightarrow A^t A \bar{x} = \bar{x}$ för alla $\bar{x}$.