3.10

$$\begin{cases} 2x_1 + x_2 + x_3 = 0 \\ 6x_1 + 3x_2 + 2x_3 = 0 \\ 4x_1 + 3x_2 + 5x_2 = 2 \end{cases} \Rightarrow [A|B] = \left[\begin{array}{ccc|c} \multicolumn{3}{c|}{\overbrace{\begin{matrix} 2 & 1 & 1 \\ 6 & 3 & 2 \\ 4 & 3 & 5 \end{matrix}}^{\text{ekvssystemmatris}}} & \begin{matrix} 0 \\ 0 \\ 2 \end{matrix} \end{array}\right] \left(\frac{\text{utökad}}{\text{matris}}\right)$$

$A \cdot X = B$, Gausselimination:

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 6 & 3 & 2 & 0 \\ 4 & 3 & 5 & 2 \end{array}\right] \times(-3), (-2) \sim \left[\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 3 & 2 \end{array}\right] \times \tfrac{1}{2} \sim \left[\begin{array}{ccc|c} 1 & \tfrac{1}{2} & \tfrac{1}{2} & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & -1 & 0 \end{array}\right] \times(-1) \sim$$

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & \tfrac{1}{2} & \tfrac{1}{2} & 0 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

bakåtsubstitution

(trappformad matris med ledande ettor)

$$\sim \begin{cases} x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_2 = 0 \\ x_2 + 3x_3 = 2 \\ x_3 = 0 \end{cases} \Rightarrow \quad x_3 = 0, \; x_2 = 2, \; x_1 = -1$$

3.11

$$\begin{cases} 2x_1 + x_2 - x_3 = -1 \\ 3x_1 - 2x_2 + x_3 = 7 \\ x_1 - 3x_2 + 2x_3 = 8 \end{cases} \qquad A \cdot X = B$$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & -1 \\ 3 & -2 & 1 & 7 \\ 1 & -3 & 2 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 2 & 1 & -1 & -1 \\ 3 & -2 & 1 & 7 \end{array}\right] \times(-2), (-3) \sim$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 7 & -5 & -17 \\ 0 & 7 & -5 & -17 \end{array}\right] \times(-1) \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 7 & -5 & -17 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right] \times \tfrac{1}{7} \sim$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & -\frac{5}{7} & -\frac{17}{7} \end{array}\right] \sim \begin{cases} x_1 - 3x_2 + 2x_3 = 8 \\ x_2 - \frac{5}{7}x_3 = -\frac{17}{7} \end{cases}$$

$$\begin{cases} x_3 = t, \quad t \in R \\ x_2 = \frac{5}{7}t - \frac{17}{7} \\ x_1 = 3\left(\frac{5}{7}t - \frac{17}{7}\right) - 2t + 8 = \frac{5}{7} + \frac{t}{7} \end{cases}$$

eller

$$\begin{cases} x_1 = \frac{5}{7} + \frac{t}{7} \\ x_2 = -\frac{17}{7} + \frac{5}{7}t \\ x_3 = t, \quad t \in R \end{cases}$$

(En geometrisk tolkning = en rät linje i rummet)

3.12

$$\begin{cases} 2x_1 + x_2 - x_3 = 2 \\ 3x_1 - 2x_2 + x_3 = 7 \\ x_1 - 3x_2 + 2x_3 = 8 \end{cases}$$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 3 & -2 & 1 & 7 \\ 1 & -3 & 2 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & 1 & 7 \end{array}\right] \times(-2), (-3) \sim$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 7 & -5 & -14 \\ 0 & 7 & -7 & -17 \end{array}\right] \times(-1) \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 7 & -5 & -14 \\ 0 & 0 & 0 & \underset{\ne 0}{-3} \end{array}\right] \sim 0 = -3$$

$\sim \emptyset$ (den tomma mängden)

inga lösningar

3.15
$$\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 0 \\ 2x_1 + 3x_2 - x_3 - 2x_4 = 0 \\ 4x_1 + 5x_2 + 3x_3 = 0 \end{cases}$$
(ett homogent system)

$$[A|B] = \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 2 & 3 & -1 & -2 & 0 \\ 4 & 5 & 3 & 0 & 0 \end{array}\right] \times(-2), (-4) \sim$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & -5 & -4 & 0 \\ 0 & 1 & -5 & -4 & 0 \end{array}\right] \times(-1) \sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & -5 & -4 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right] \sim$$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & -5 & -4 & 0 \end{array}\right] \end{array} \sim \begin{cases} x_1 + x_2 + 2x_3 + x_4 = 0 \\ x_2 - 5x_3 - 4x_4 = 0 \end{cases}$$

$x_4 = t,\ x_3 = s,\ x_2 = 5s + 4t,\ x_1 = -(5s+4t) - 2s - t =$

$= -7s - 5t$ eller

$$\begin{cases} x_1 = -7s - 5t \\ x_2 = 5s + 4t \\ x_3 = s \\ x_4 = t, \quad \underline{t, s \in R} \end{cases} \text{ eller } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = s \begin{pmatrix} -7 \\ 5 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ 4 \\ 0 \\ 1 \end{pmatrix}$$

(En geometrisk tolkning = ett plan i R^4)

3.25 Bestäm X s.a. $AX + XA = B$, där

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \text{o} \quad B = \begin{bmatrix} 6 & -2 \\ 2 & 0 \end{bmatrix}$$

Obs X är en (2×2) matris

Sätt $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$. Bestäm x_i, $i=1,\dots,4$.

Räkna $AX = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} (x_1+2x_3) & (x_2+2x_4) \\ x_3 & x_4 \end{bmatrix}$

$XA = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 & (2x_1+x_2) \\ x_3 & (2x_3+x_4) \end{bmatrix}$

$AX + XA = \begin{bmatrix} (2x_1+2x_3) & (2x_1+2x_2+2x_4) \\ 2x_3 & (2x_3+2x_4) \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 2 & 0 \end{bmatrix}$

$$\Rightarrow \begin{cases} 2x_1 + 2x_3 = 6 \\ 2x_1 + 2x_2 + 2x_4 = -2 \\ 2x_3 = 2 \\ 2x_3 + 2x_4 = 0 \end{cases} \Leftrightarrow \quad \begin{matrix} x_3 = 1, & x_4 = -1 \\ x_1 = 2, & x_2 = -2 \end{matrix}$$

$\Rightarrow X = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

3.33 $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

(a) Bestäm alla X s.a. $AX = Y$ där $Y = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

$$[A|Y] = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ -1 & 2 & 0 & -2 \\ 0 & 2 & -1 & -1 \end{array}\right] \times(1) \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 2 & -1 & -1 \\ 0 & 2 & -1 & -1 \end{array}\right] \times(-1) \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 2 & -1 & -1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right] \end{array} \sim$$

$$\begin{cases} x_1 - x_3 = 1 \\ x_2 - \frac{1}{2}x_3 = -\frac{1}{2} \end{cases}$$

$x_3 = t,\ t \in R$

$x_2 = \frac{1}{2}t - \frac{1}{2},\quad x_1 = 1 + t$

eller $\begin{cases} x_1 = 1+t \\ x_2 = -\frac{1}{2} + \frac{1}{2}t \\ x_3 = t, \quad t \in R \end{cases}$ (en geometrisk tolkning = en rät linje)

<u>3.47</u> Lös i minstakvadrat mening:

$$\begin{cases} 2x_1 + 2x_2 = 6 \\ 3x_1 - x_2 = 1 \\ 4x_1 + 2x_2 = 9 \end{cases}$$

Systemet på matrisform $AX = B$ där

$$A = \begin{bmatrix} 2 & 2 \\ 3 & -1 \\ 4 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 1 \\ 9 \end{bmatrix}$$

<u>Normalekvationerna:</u> $A^t \cdot A \cdot X = A^t B \quad (*)$

$$A^t A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 3 & -1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} (4+9+16) & (4-3+8) \\ (4-3+8) & (4+1+4) \end{bmatrix} =$$

$$= \begin{bmatrix} 29 & 9 \\ 9 & 9 \end{bmatrix}, \quad A^t B = \begin{bmatrix} 2 & 3 & 4 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \\ 9 \end{bmatrix} = \begin{bmatrix} 12+3+36 \\ 12-1+18 \end{bmatrix} = \begin{bmatrix} 51 \\ 29 \end{bmatrix}$$

Lös (*): $\begin{bmatrix} 29 & 9 & | & 51 \\ 9 & 9 & | & 29 \end{bmatrix} \begin{matrix} \leftarrow \\ \times(-1) \end{matrix} \sim \begin{bmatrix} 20 & 0 & | & 22 \\ 9 & 9 & | & 29 \end{bmatrix} \begin{matrix} \times\frac{1}{20} \\ \times\frac{1}{9} \end{matrix} \sim$

$$\begin{array}{cc|c} x_1 & x_2 & \\ \hline \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{11}{10} \\ 1 & 1 & \frac{29}{9} \end{array}\right] \sim \quad x_1 = \frac{11}{10}$$

$$x_1 + x_2 = \frac{29}{9} \Rightarrow x_2 = \frac{29}{9} - \frac{11}{10} =$$

$$= \frac{290-99}{90} = \frac{191}{90}$$

3.48 $\begin{cases} 2x_1 - x_2 = 2 \\ -2x_1 + x_2 = 2 \\ 4x_1 - 2x_2 = 0 \end{cases}$ $AX = B$, där

$$A = \begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

(*): $A^t \cdot A \cdot X = A^t B$;

$$A^t \cdot A = \begin{bmatrix} 2 & -2 & 4 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -2 & 1 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} (4+4+16) & (-2-2-8) \\ (-2-2-8) & (1+1+4) \end{bmatrix} =$$

$$= \begin{bmatrix} 24 & -12 \\ -12 & 6 \end{bmatrix}, \quad A^t B = \begin{bmatrix} 2 & -2 & 4 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 4-4+0 \\ -2+2+0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Lös (*): $\left[\begin{array}{cc|c} 24 & -12 & 0 \\ -12 & 6 & 0 \end{array}\right] \sim \begin{array}{ccc} x_1 & x_2 & \\ \hline \end{array} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \end{array}\right] \sim$

$$x_1 - \frac{1}{2}x_2 = 0 \Leftrightarrow \begin{cases} x_2 = t \\ x_1 = \frac{1}{2}t \end{cases} \quad \text{eller} \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = t\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix},$$

$t \in \mathbb{R}$

3.50 Data

x	1	2	3	4	5
y	2	5	6	8	11

Bestäm den räta linje som approximerar datan i minstakvadrat mening.

$y = k\cdot x + b$ vi skall hitta k, b.

Motsvarande system;
$$\begin{cases} 2 = k\cdot 1 + b \\ 5 = k\cdot 2 + b \\ 6 = k\cdot 3 + b \\ 8 = k\cdot 4 + b \\ 11 = k\cdot 5 + b \end{cases}$$
eller $AX = B$ där

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 8 \\ 11 \end{bmatrix}$$

Normalekvationerna: $A^t A\cdot X = A^t B$, $X = \begin{bmatrix} k \\ b \end{bmatrix}$

$$A^t A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} (1+4+9+16+25) & (1+2+3+4+5) \\ (1+2+3+4+5) & (1+1+1+1+1) \end{bmatrix}$$

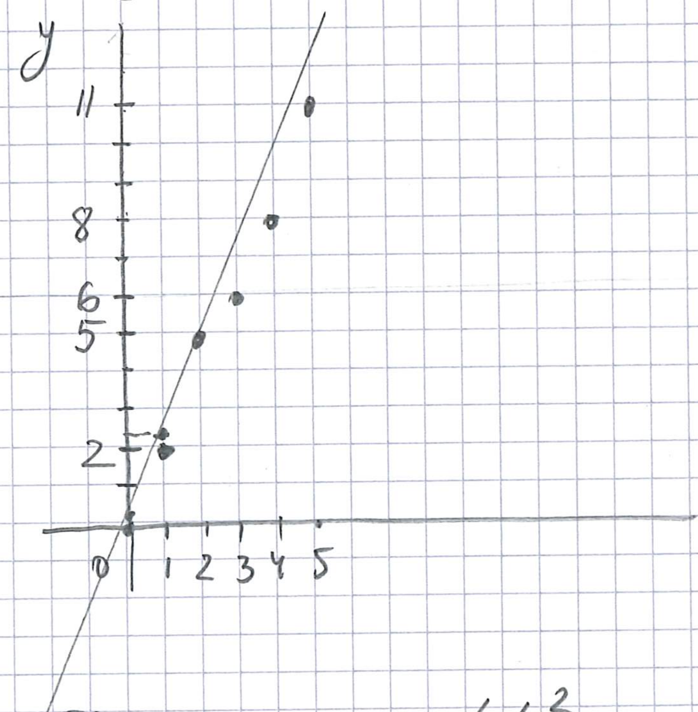
$$= \begin{bmatrix} 55 & 15 \\ 15 & 5 \end{bmatrix}$$

$$A^t B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \\ 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 2+10+18+32+55 \\ 2+5+6+8+11 \end{bmatrix} = \begin{bmatrix} 117 \\ 32 \end{bmatrix}$$

Lös systemet: $\left[\begin{array}{cc|c} 55 & 15 & 117 \\ 15 & 5 & 32 \end{array}\right] \times(-3) \sim$

$$\begin{array}{cc} k & b \\ \end{array}$$
$$\left[\begin{array}{cc|c} 10 & 0 & 21 \\ 15 & 5 & 32 \end{array}\right] \sim \begin{cases} 10k = 21 \\ 15k + 5b = 32 \end{cases} \qquad \begin{array}{l} k = 2.1 \\ b = 0.1 \end{array}$$

$\Rightarrow \quad y = 2.1 \cdot x + 0.1$



3.53 $\quad y = a + bt^2 \quad$ typen av sambandet mellan t o y.

Data:

t	0	0.2	0.4	0.6	0.8	1.0
y	2.1	2.5	3.5	5.5	8.5	12.2

Ekvationen: $\quad A \cdot X = B \quad$ där

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 0.04 \\ 1 & 0.16 \\ 1 & 0.36 \\ 1 & 0.64 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2.1 \\ 2.5 \\ 3.5 \\ 5.5 \\ 8.5 \\ 12.2 \end{bmatrix}. \quad X = \begin{bmatrix} a \\ b \end{bmatrix}.$$

$$A^tAX = A^tB$$

$$A^tA = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.04 & 0.16 & 0.36 & 0.64 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0.04 \\ 1 & 0.16 \\ 1 & 0.36 \\ 1 & 0.64 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2.2 \\ 2.2 & 1.5664 \end{bmatrix}$$

$$A^tB = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.04 & 0.16 & 0.36 & 0.64 & 1 \end{bmatrix} \begin{bmatrix} 2.1 \\ 2.5 \\ 3.5 \\ 5.5 \\ 8.5 \\ 12.2 \end{bmatrix} = \begin{bmatrix} 34.3 \\ 20.28 \end{bmatrix}$$

Lös systemet:

$$\begin{array}{cc} a & b \\ \end{array}$$
$$\left[\begin{array}{cc|c} 6 & 2.2 & 34.3 \\ 2.2 & 1.5664 & 20.28 \end{array}\right]$$

$\Rightarrow \quad a \approx 2.0, \; b \simeq 10.1$