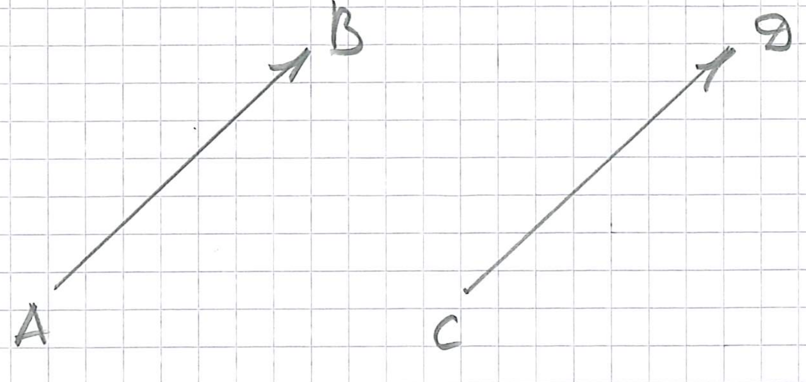
Vektorer



Beteckning: $\overline{AB}$, $\overline{u}$

Obs $\overline{u} = \overline{v} \iff \overline{u}, \overline{v}$ har samma
längd o samma riktning.

Operationer på vektorer:

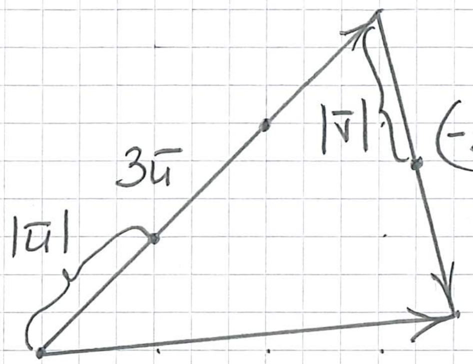
- multiplikation med ett tal: $\overline{u}, t \in R \longrightarrow t \cdot \overline{u}$

- addition: $\overline{u}, \overline{v} \longrightarrow \overline{u} + \overline{v}$

EX 1 Låt $\bar{u} =$  $= \bar{v}$

Finn $\bar{w} = 3\bar{u} - 2\bar{v}$

Obs $\bar{w} = 3\bar{u} + (-2)\cdot\bar{v}$



$(-2)\cdot\bar{v} = -2\bar{v}$ (Obs riktningen)

Egenskaper:

$\bar{u}+\bar{v} = \bar{v}+\bar{u}$, $(\bar{u}+\bar{v})+\bar{w} = \bar{u}+(\bar{v}+\bar{w})$, $\bar{u}+\bar{0} = \bar{u}$,

$\bar{u}+(-\bar{u}) = \bar{0}$, $(t+s)\bar{u} = t\bar{u}+s\bar{u}$, $t\cdot(\bar{u}+\bar{v}) = t\bar{u}+t\bar{v}$,

$t(s\cdot\bar{u}) = (ts)\bar{u}$, $1\cdot\bar{u} = \bar{u}$.

EX 2 $5\bar{u} + 2\bar{v} - 3\bar{u} + 7\bar{v} =$

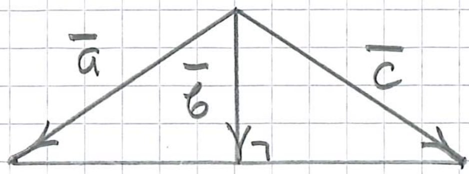
$= (5\bar{u} - 3\bar{u}) + (2\bar{v} + 7\bar{v}) = (5-3)\cdot\bar{u} + (2+7)\bar{v} =$

$= 2\bar{u} + 9\bar{v}$.

En linjär kombination av vektorer:

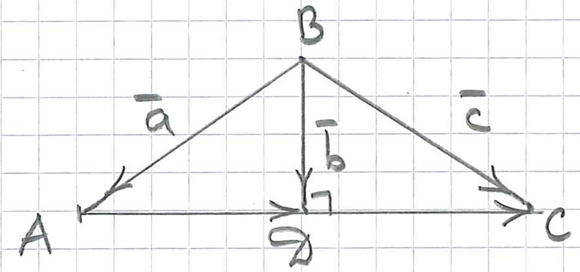
$$\lambda_1 \cdot \bar{u}_1 + \lambda_2 \cdot \bar{u}_2 + \dots + \lambda_n \cdot \bar{u}_n$$

EX 3.



Låt $|\bar{a}| = |\bar{c}|$. Uttryck $\bar{c}$ som en linjär kombination av $\bar{a}$ o $\bar{b}$.

Lsg.



Obs $\bar{c} = \bar{b} + \overline{DC}$, $\overline{DC} = \overline{AD}$, $\bar{a} + \overline{AD} = \bar{b}$

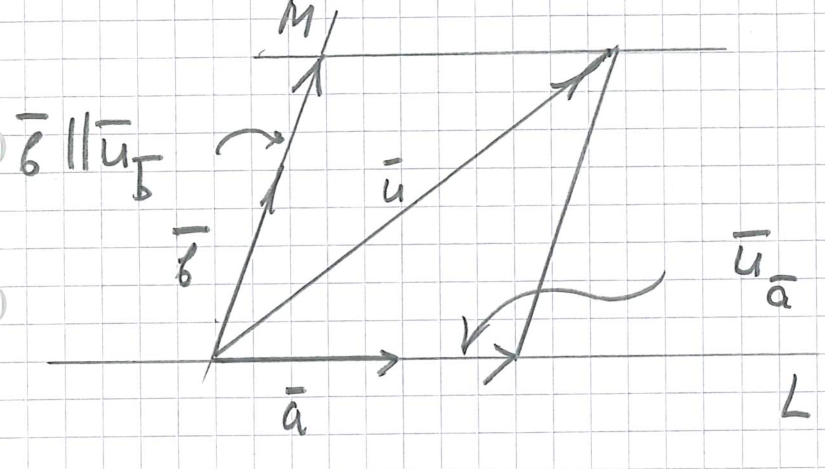
Då är $\overline{AD} = \bar{b} - \bar{a}$ o $\bar{c} = \bar{b} + (\bar{b} - \bar{a}) =$

$= 2\bar{b} - \bar{a} = \underline{(-1) \cdot \bar{a} + 2 \cdot \bar{b}}$

En bas i planet: $\bar{a}, \bar{b}$ s.a. $\bar{a} \nparallel \bar{b}$

Egenskaper:

varje vektor $\bar{u}$ i planet kan skrivas som lin. kombination av basvektorerna på entydigt sätt!



$$\bar{u} = \bar{u}_{\bar{a}} + \bar{u}_{\bar{b}} = x \cdot \bar{a} + y \cdot \bar{b}$$

Koordinater av $\bar{u}$ i basen $\bar{a}, \bar{b}$.

$\bar{u} = \begin{pmatrix} x \\ y \end{pmatrix}$ (identifiering)

- $\lambda \cdot \bar{u} = \lambda \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \lambda \cdot x \\ \lambda \cdot y \end{pmatrix}$
- $\bar{u}_1 + \bar{u}_2 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$

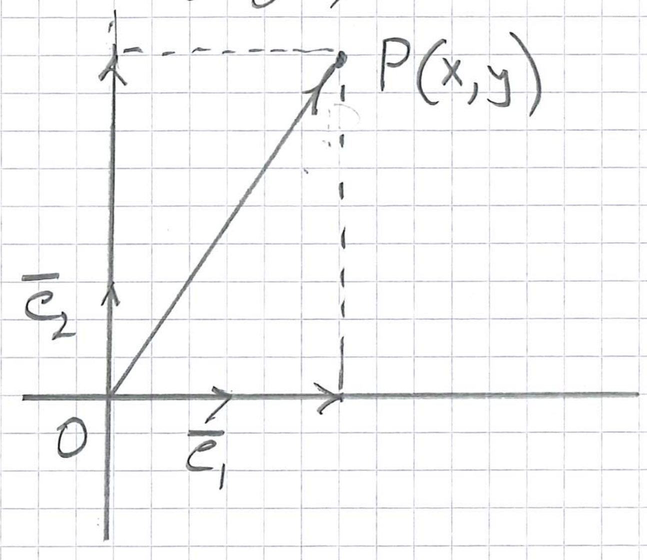
Operationer på koordinatform.

EX 4. $\bar{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\bar{v} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. Finn $\bar{w} = 2\bar{u} + 3\bar{v}$

$$\bar{w} = 2 \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ -9 \end{pmatrix} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}.$$

Ett ortonormerat koordinatsystem i planet.

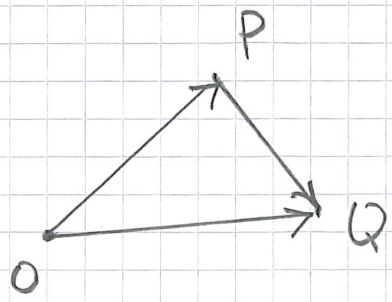
O (origo) o en ON-bas $\bar{e}_1, \bar{e}_2$ $\left(\bar{e}_1 \perp \bar{e}_2 \text{ o } |\bar{e}_1| = |\bar{e}_2| = 1 \right)$



$$\overline{OP} = x \cdot \bar{e}_1 + y \cdot \bar{e}_2$$

EX 5. Låt $P(2,3)$, $Q(4,-5)$.

Finn $\overline{PQ}$ samt $|\overline{PQ}|$.



Obs $\overline{PQ} = \overline{OQ} - \overline{OP} =$

$$= \begin{pmatrix} 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

$$|\overline{PQ}| = \sqrt{2^2 + (-8)^2} = 2 \cdot \sqrt{1 + 4^2} = \underline{2 \cdot \sqrt{17}}$$