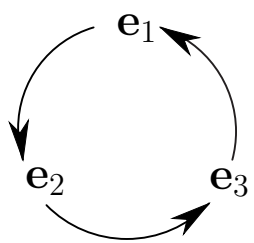
Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



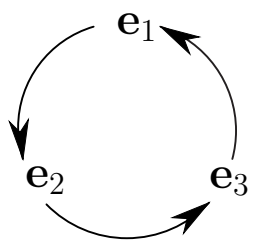
$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



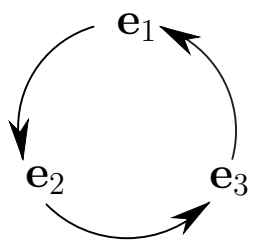
$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3)$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

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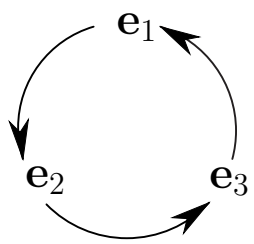
$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_1(\mathbf{e}_1 \times \mathbf{e}_1) + x_1y_2(\mathbf{e}_1 \times \mathbf{e}_2) + x_1y_3(\mathbf{e}_1 \times \mathbf{e}_3)$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

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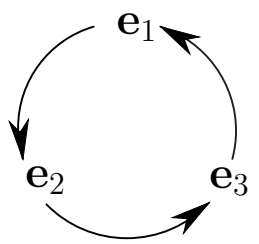
$$= x_1y_1(\mathbf{e}_1 \times \mathbf{e}_1) + x_1y_2(\mathbf{e}_1 \times \mathbf{e}_2) + x_1y_3(\mathbf{e}_1 \times \mathbf{e}_3) +$$

$$+ x_2y_1(\mathbf{e}_2 \times \mathbf{e}_1) + x_2y_2(\mathbf{e}_2 \times \mathbf{e}_2) + x_2y_3(\mathbf{e}_2 \times \mathbf{e}_3)$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

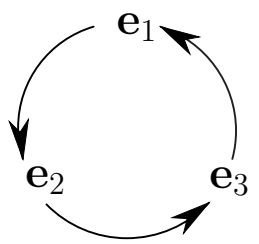
$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_1(\mathbf{e}_1 \times \mathbf{e}_1) + x_1y_2(\mathbf{e}_1 \times \mathbf{e}_2) + x_1y_3(\mathbf{e}_1 \times \mathbf{e}_3) +$$

$$+ x_2y_1(\mathbf{e}_2 \times \mathbf{e}_1) + x_2y_2(\mathbf{e}_2 \times \mathbf{e}_2) + x_2y_3(\mathbf{e}_2 \times \mathbf{e}_3) +$$

$$+ x_3y_1(\mathbf{e}_3 \times \mathbf{e}_1) + x_3y_2(\mathbf{e}_3 \times \mathbf{e}_2) + x_3y_3(\mathbf{e}_3 \times \mathbf{e}_3)$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

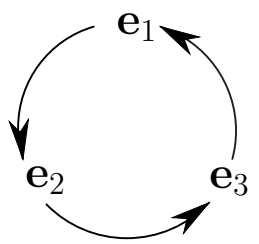
$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_1 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_1)}_{=\mathbf{0}} + x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_2 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_2)}_{=\mathbf{0}} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} + x_3y_3 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_3)}_{=\mathbf{0}} =$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

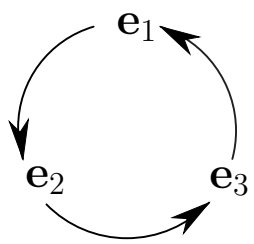
Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$\begin{aligned} = \quad & x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} + \\ + \ & x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + \quad x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} + \\ + \ & x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} \quad = \end{aligned}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

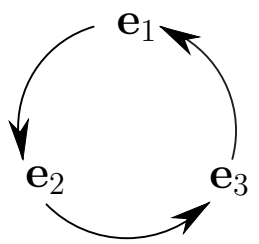
$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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Moturs *Medurs*

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$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

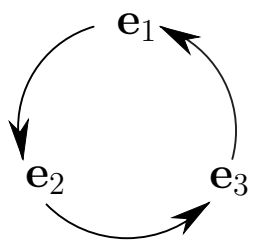
$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

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$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

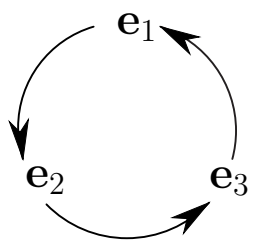
$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

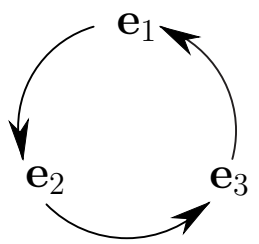
$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$x_1 \qquad y_1$$

$$x_2 \qquad y_2$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

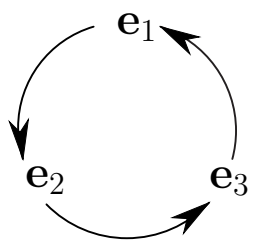
$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 \\ x_2 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_1 \\ y_2 \end{pmatrix} =$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

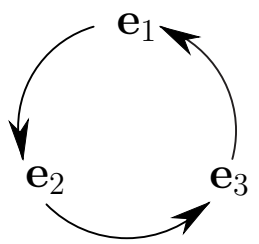
$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ \\ \\ \end{pmatrix}$$

$$\begin{matrix} x_1 & y_1 \\ x_2 & y_2 \end{matrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

$$\mathbf{e}_1 \times \mathbf{e}_1 = \mathbf{e}_2 \times \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_3 = \mathbf{0}$$

$$\mathbf{u} \times \mathbf{v} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3) \times (y_1\mathbf{e}_1 + y_2\mathbf{e}_2 + y_3\mathbf{e}_3) =$$

$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

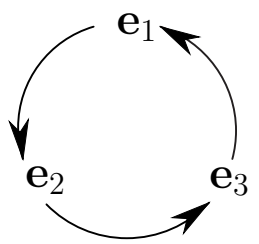
$$+ x_2y_1 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_1)}_{=-\mathbf{e}_3} + x_2y_3 \underbrace{(\mathbf{e}_2 \times \mathbf{e}_3)}_{=\mathbf{e}_1} +$$

$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

$$= (x_2y_3 - x_3y_2)\mathbf{e}_1 + (x_3y_1 - x_1y_3)\mathbf{e}_2 + (x_1y_2 - x_2y_1)\mathbf{e}_3 =$$

$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 \\ x_2 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_1 \\ y_2 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ \\ \\ \end{pmatrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



$$\Longrightarrow \quad \begin{cases} \mathbf{e}_1 \times \mathbf{e}_2 = \mathbf{e}_3 \ , & \mathbf{e}_2 \times \mathbf{e}_1 = -\mathbf{e}_3 \\ \mathbf{e}_2 \times \mathbf{e}_3 = \mathbf{e}_1 \ , & \mathbf{e}_3 \times \mathbf{e}_2 = -\mathbf{e}_1 \\ \mathbf{e}_3 \times \mathbf{e}_1 = \mathbf{e}_2 \ , & \mathbf{e}_1 \times \mathbf{e}_3 = -\mathbf{e}_2 \end{cases}$$

Moturs *Medurs*

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$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

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$$+ x_3y_1 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_1)}_{=\mathbf{e}_2} + x_3y_2 \underbrace{(\mathbf{e}_3 \times \mathbf{e}_2)}_{=-\mathbf{e}_1} =$$

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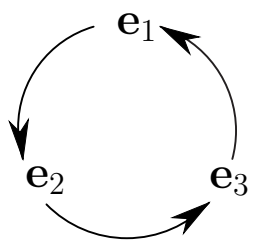
$$= \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_1 \\ x_2 \end{pmatrix} \times \underline{\mathbf{e}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_1 \\ y_2 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ \\ \end{pmatrix}$$

Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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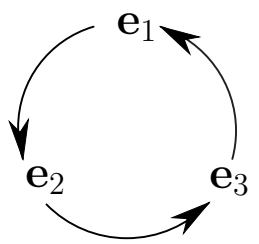
Beräkning av kryssprodukten i högerorienterad ON-bas, boken sid 37



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Moturs *Medurs*

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$$= x_1y_2 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_2)}_{=\mathbf{e}_3} + x_1y_3 \underbrace{(\mathbf{e}_1 \times \mathbf{e}_3)}_{=-\mathbf{e}_2} +$$

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