

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right)$$

$A \quad X = \quad I$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1}$$

$A \quad X = \quad I$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right)$$

$A \quad X = \quad I$        $A_1 \quad X = \quad Y_1$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right)$$

$A \qquad X = \qquad I$        $A_1 \qquad X = \qquad Y_1$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$\sim$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\textcolor{red}{r_3+r_2}}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$

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$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ \color{red}{0} & \color{red}{-1} & \color{red}{1} & \color{red}{1} & \color{red}{1} & \color{red}{0} \\ \color{red}{0} & \color{red}{1} & \color{red}{2} & \color{red}{0} & \color{red}{0} & \color{red}{1} \end{array} \right) \xrightarrow{\color{red}{r_3+r_2}} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ \color{red}{0} & \color{red}{-1} & \color{red}{1} & \color{red}{1} & \color{red}{1} & \color{red}{0} \\ \color{red}{0} & \color{red}{1} & \color{red}{2} & \color{red}{0} & \color{red}{0} & \color{red}{1} \end{array} \right)$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ \color{red}{0} & \color{red}{0} & \color{red}{3} & \color{red}{1} & \color{red}{1} & \color{red}{1} \end{array} \right)$$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$$\sim \left( \begin{array}{ccc|ccc} \color{red}{1} & \color{red}{-1} & \color{red}{0} & \color{red}{1} & \color{red}{0} & \color{red}{0} \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[\color{black}{-3r_2}]{\color{red}{3r_1}}$$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} \color{red}{3} & \color{red}{-3} & \color{red}{0} & \color{red}{3} & \color{red}{0} & \color{red}{0} \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ \textcolor{red}{0} & \textcolor{red}{-1} & \textcolor{red}{1} & \textcolor{red}{1} & \textcolor{red}{1} & \textcolor{red}{0} \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[\textcolor{red}{-3r_2}]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$

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$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_2+r_3}$$

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# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$A \quad X = \quad I \qquad A_1 \quad X = \quad Y_1$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[r_2+r_3]$$

$A_2 \quad X = \quad Y_2 \qquad A_3 \quad X = \quad Y_3$

$$\sim \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$

$A_4 \quad X = \quad Y_4$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$A \quad X = \quad I \qquad A_1 \quad X = \quad Y_1$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_2+r_3}$$

$A_2 \quad X = \quad Y_2 \qquad A_3 \quad X = \quad Y_3$

$$\sim \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_1+r_2}$$

$A_4 \quad X = \quad Y_4$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$
$$A \quad X = \quad I \qquad A_1 \quad X = \quad Y_1$$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_2+r_3}$$
$$A_2 \quad X = \quad Y_2 \qquad A_3 \quad X = \quad Y_3$$

$$\sim \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_1+r_2} \left( \begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & -2 & 1 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$
$$A_4 \quad X = \quad Y_4 \qquad 3IX = \quad 3X = \quad Y_5$$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$A \quad X = \quad I \qquad A_1 \quad X = \quad Y_1$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_2+r_3}$$

$A_2 \quad X = \quad Y_2 \qquad A_3 \quad X = \quad Y_3$

$$\sim \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_1+r_2} \left( \begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & -2 & 1 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right)$$

$A_4 \quad X = \quad Y_4 \qquad 3IX = \quad 3X = \quad Y_5$

# Matrisinvers

$$\left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_2+r_1} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{r_3+r_2}$$

$A \quad X = \quad I \qquad A_1 \quad X = \quad Y_1$

$$\sim \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[-3r_2]{3r_1} \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & -3 & -3 & -3 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_2+r_3}$$

$A_2 \quad X = \quad Y_2 \qquad A_3 \quad X = \quad Y_3$

$$\sim \left( \begin{array}{ccc|ccc} 3 & -3 & 0 & 3 & 0 & 0 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow{r_1+r_2} \left( \begin{array}{ccc|ccc} 3 & 0 & 0 & 1 & -2 & 1 \\ 0 & 3 & 0 & -2 & -2 & 1 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array} \right) \xrightarrow[r_3/3]{r_1/3, r_2/3}$$

$A_4 \quad X = \quad Y_4 \qquad 3IX = \quad 3X = \quad Y_5$

# Matrisinvers

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array} \right)$$
$$I \qquad X = \qquad A^{-1}$$

# Matrisinvers

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array} \right)$$
$$I \qquad X = \qquad A^{-1}$$

Eftersom lösningen till ekvationen är  $X = A^{-1}$  (om den existerar, vilket den gör enligt vår kalkyl) följer det att

# Matrisinvers

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array} \right)$$
$$I \qquad X = \qquad A^{-1}$$

Eftersom lösningen till ekvationen är  $X = A^{-1}$  (om den existerar, vilket den gör enligt vår kalkyl) följer det att

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & 1/3 \\ -2/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & -2 & 1 \\ -2 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

# Matrisinvers

$$\sim \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & -2/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array} \right)$$
$$I \qquad X = \qquad A^{-1}$$

Eftersom lösningen till ekvationen är  $X = A^{-1}$  (om den existerar, vilket den gör enligt vår kalkyl) följer det att

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & 1/3 \\ -2/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & -2 & 1 \\ -2 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Kontrollera nu själv att

$$A \cdot A^{-1} = I \qquad \text{och} \qquad A^{-1} \cdot A = I.$$