

Banta ner och fyll ut.

$$\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 =$$

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$$\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) =$$

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$$\begin{aligned}\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\ &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix}\end{aligned}$$

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$$\begin{aligned}\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\&= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}\end{aligned}$$

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$$\begin{aligned}\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\&= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}\end{aligned}$$

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$$\begin{aligned}
 \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \left(\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & a_0 \\ -1 & 0 & -2 & 0 & a_1 \\ 1 & -1 & 3 & 0 & a_2 \\ -1 & 0 & -2 & 0 & a_3 \end{array} \right) \sim
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 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ -1 & 0 & -2 & | & 0 & a_1 \\ 1 & -1 & 3 & | & 0 & a_2 \\ -1 & 0 & -2 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & -2 & 2 & | & 0 & a_2 - a_0 \\ 0 & 1 & -1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim
 \end{aligned}$$

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$$\begin{aligned}
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& = \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
& \iff \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ -1 & 0 & -2 & | & 0 & a_1 \\ 1 & -1 & 3 & | & 0 & a_2 \\ -1 & 0 & -2 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & -2 & 2 & | & 0 & a_2 - a_0 \\ 0 & 1 & -1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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 \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ -1 & 0 & -2 & | & 0 & a_1 \\ 1 & -1 & 3 & | & 0 & a_2 \\ -1 & 0 & -2 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & -2 & 2 & | & 0 & a_2 - a_0 \\ 0 & 1 & -1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
 \end{aligned}$$

Beroendeekvationen: $\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}$

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$$\begin{aligned}
 \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ -1 & 0 & -2 & | & 0 & a_1 \\ 1 & -1 & 3 & | & 0 & a_2 \\ -1 & 0 & -2 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & -2 & 2 & | & 0 & a_2 - a_0 \\ 0 & 1 & -1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
 \end{aligned}$$

Beroendeekvationen: $\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R} \implies -2\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0}$

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$$\begin{aligned}
 \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 + \lambda_3 \mathbf{p}_3 &= \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) + \lambda_3 (1 - 2x + 3x^2 - 2x^3) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
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 \end{aligned}$$

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 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -2 \\ 3 \\ -2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 3 \\ -1 & 0 & -2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ -1 & 0 & -2 & | & 0 & a_1 \\ 1 & -1 & 3 & | & 0 & a_2 \\ -1 & 0 & -2 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & -2 & 2 & | & 0 & a_2 - a_0 \\ 0 & 1 & -1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 0 & a_0 \\ 0 & 1 & -1 & | & 0 & a_1 + a_0 \\ 0 & 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
 \end{aligned}$$

Beroendeekvationen: $\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R} \implies -2\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0} \iff \mathbf{p}_3 = 2\mathbf{p}_1 - \mathbf{p}_2$

Utse \mathbf{p}_3 till löjligt element, dvs stryk \mathbf{p}_3 och börja om från början.

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$$\begin{aligned}
 & \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) = \\
 & = \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 & \iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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 & \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 & \iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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För att hitta utfyllnad behöver \mathbb{U} uttryckas som lösningsrum.

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$$\begin{aligned}
 & \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 & \iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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$$\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \text{ lösbart omm}$$

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$$\begin{aligned}
 & \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
 \end{aligned}$$

Beroendeekvationen: $\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, $t \in \mathbb{R} \implies -2\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0} \iff \mathbf{p}_3 = 2\mathbf{p}_1 - \mathbf{p}_2$

Utse \mathbf{p}_3 till löjligt element, dvs stryk \mathbf{p}_3 och börja om från början. Ger $\mathbf{p}_1, \mathbf{p}_2$ linjärt oberoende.

För att hitta utfyllnad behöver \mathbb{U} uttryckas som lösningsrum.

$$\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \text{ lösbart omm } \begin{cases} a_0 + 2a_1 + a_2 = 0 \\ -a_1 + a_3 = 0 \end{cases}$$

Banta ner och fyll ut.

$$\begin{aligned}
 & \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = \lambda_1 (1 - x + x^2 - x^3) + \lambda_2 (1 - x^2) = \\
 &= \lambda_1 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \underline{\mathbf{x}} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \underline{\mathbf{x}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ resp } \mathbf{q} = \underline{\mathbf{x}} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \iff \\
 &\iff \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ -1 & 0 & | & 0 & a_1 \\ 1 & -1 & | & 0 & a_2 \\ -1 & 0 & | & 0 & a_3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & -2 & | & 0 & a_2 - a_0 \\ 0 & 1 & | & 0 & a_3 + a_0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & | & 0 & a_0 \\ 0 & 1 & | & 0 & a_1 + a_0 \\ 0 & 0 & | & 0 & a_0 + 2a_1 + a_2 \\ 0 & 0 & | & 0 & -a_1 + a_3 \end{pmatrix}.
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Beroendeekvationen: $\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = t \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, $t \in \mathbb{R} \implies -2\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0} \iff \mathbf{p}_3 = 2\mathbf{p}_1 - \mathbf{p}_2$

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För att hitta utfyllnad behöver \mathbb{U} uttryckas som lösningsrum.

$$\lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \text{ lösbart omm } \begin{cases} a_0 + 2a_1 + a_2 = 0 \\ -a_1 + a_3 = 0 \end{cases} \iff$$

$$\mathbb{U} = [\mathbf{p}_1, \mathbf{p}_2] = \left\{ \mathbf{q} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \in \mathbb{P}_3 : \begin{array}{l} a_0 + 2a_1 + a_2 = 0 \\ -a_1 + a_3 = 0 \end{array} \right\}$$