

Andragradsyta med 1:a gradstermer.

$$Q(\mathbf{u}) = Q(\underline{\mathbf{e}} X) = Q \left(\underline{\mathbf{e}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \right) = 2x_1x_2 + 2x_1x_3 + 2x_2x_3 = X^t \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} X,$$

$$\mathbf{f}_1 = \frac{1}{\sqrt{2}} \underline{\mathbf{e}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}_{\lambda=-1}, \quad \mathbf{f}_2 = \frac{1}{\sqrt{6}} \underline{\mathbf{e}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}_{\lambda=-1}, \quad \mathbf{f}_3 = \frac{1}{\sqrt{3}} \underline{\mathbf{e}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}_{\lambda=2}$$

$$\underline{\mathbf{f}} = \underline{\mathbf{e}} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \implies Q(\mathbf{u}) = Q(\underline{\mathbf{f}} Y) = Q \left(\underline{\mathbf{f}} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \right) = -y_1^2 - y_2^2 + 2y_3^2$$

$$g(x_1,x_2,x_3)=Q(\mathbf{u})-6x_1-6x_2-4x_3=$$

$$g(x_1,x_2,x_3) = Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \left(\begin{array}{ccc} -6 & -6 & -4 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right)$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \frac{16}{\sqrt{3}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \begin{pmatrix} -\frac{2}{\sqrt{2}} & -\frac{2}{\sqrt{6}} & \frac{16}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \textcolor{red}{y}_1 \\ \textcolor{blue}{y}_2 \\ \textcolor{magenta}{y}_3 \end{pmatrix} = \\
&= -\left(y_1^2 - \frac{2}{\sqrt{2}}y_1\right) - \left(y_2^2 - \frac{2}{\sqrt{6}}y_2\right) + 2\left(y_3^2 - \frac{8}{\sqrt{3}}y_2\right)
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \frac{16}{\sqrt{3}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -\left(y_1^2 - \frac{2}{\sqrt{2}}y_1 \right) - \left(y_2^2 - \frac{2}{\sqrt{6}}y_2 \right) + 2\left(y_3^2 - \frac{8}{\sqrt{3}}y_2 \right) = \\
&= -\left(\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \frac{1}{2} \right) - \left(\left(y_2 - \frac{1}{\sqrt{6}} \right)^2 - \frac{1}{6} \right) + 2\left(\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - \frac{16}{3} \right)
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \frac{16}{\sqrt{3}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -\left(y_1^2 - \frac{2}{\sqrt{2}}y_1 \right) - \left(y_2^2 - \frac{2}{\sqrt{6}}y_2 \right) + 2\left(y_3^2 - \frac{8}{\sqrt{3}}y_2 \right) = \\
&= -\left(\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \frac{1}{2} \right) - \left(\left(y_2 - \frac{1}{\sqrt{6}} \right)^2 - \frac{1}{6} \right) + 2\left(\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - \frac{16}{3} \right) = \\
&= -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 + \left(\frac{1}{2} + \frac{1}{6} - \frac{32}{3} \right)
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \frac{16}{\sqrt{3}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -\left(y_1^2 - \frac{2}{\sqrt{2}}y_1 \right) - \left(y_2^2 - \frac{2}{\sqrt{6}}y_2 \right) + 2\left(y_3^2 - \frac{8}{\sqrt{3}}y_2 \right) = \\
&= -\left(\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \frac{1}{2} \right) - \left(\left(y_2 - \frac{1}{\sqrt{6}} \right)^2 - \frac{1}{6} \right) + 2\left(\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - \frac{16}{3} \right) = \\
&= -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 + \left(\frac{1}{2} + \frac{1}{6} - \frac{32}{3} \right) = \\
&= -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - 10 = -9
\end{aligned}$$

$$\begin{aligned}
g(x_1, x_2, x_3) &= Q(\mathbf{u}) - 6x_1 - 6x_2 - 4x_3 = Q(\mathbf{u}) + \begin{pmatrix} -6 & -6 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \\
&= Q(\mathbf{u}) - \begin{pmatrix} 6 & 6 & 4 \end{pmatrix} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -y_1^2 - y_2^2 + 2y_3^2 - \left(-\frac{2}{\sqrt{2}} \quad -\frac{2}{\sqrt{6}} \quad \frac{16}{\sqrt{3}} \right) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \\
&= -\left(y_1^2 - \frac{2}{\sqrt{2}}y_1 \right) - \left(y_2^2 - \frac{2}{\sqrt{6}}y_2 \right) + 2\left(y_3^2 - \frac{8}{\sqrt{3}}y_2 \right) = \\
&= -\left(\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \frac{1}{2} \right) - \left(\left(y_2 - \frac{1}{\sqrt{6}} \right)^2 - \frac{1}{6} \right) + 2\left(\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - \frac{16}{3} \right) = \\
&= -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 + \left(\frac{1}{2} + \frac{1}{6} - \frac{32}{3} \right) = \\
&= -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 - 10 = -9 \iff \\
&\iff -\left(y_1 - \frac{1}{\sqrt{2}} \right)^2 - \left(y_2 - \frac{1}{\sqrt{6}} \right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}} \right)^2 = 1
\end{aligned}$$

Ekvationen

$$-\left(y_1 - \frac{1}{\sqrt{2}}\right)^2 - \left(y_2 - \frac{1}{\sqrt{6}}\right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}}\right)^2 = 1$$

definierar en två-mantlad hyperboloid med medelpunkt P där

$$\overline{OP} = \underline{\mathbf{f}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix}$$

Ekvationen

$$-\left(y_1 - \frac{1}{\sqrt{2}}\right)^2 - \left(y_2 - \frac{1}{\sqrt{6}}\right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}}\right)^2 = 1$$

definierar en två-mantlad hyperboloid med medelpunkt P där

$$\overline{OP} = \underline{\mathbf{f}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = \left[\underline{\mathbf{f}} = \underline{\mathbf{e}} T \right] =$$

Ekvationen

$$-\left(y_1 - \frac{1}{\sqrt{2}}\right)^2 - \left(y_2 - \frac{1}{\sqrt{6}}\right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}}\right)^2 = 1$$

definierar en två-mantlad hyperboloid med medelpunkt P där

$$\overline{OP} = \underline{\mathbf{f}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = [\underline{\mathbf{f}} = \underline{\mathbf{e}} T] = \underline{\mathbf{e}} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix}$$

Ekvationen

$$-\left(y_1 - \frac{1}{\sqrt{2}}\right)^2 - \left(y_2 - \frac{1}{\sqrt{6}}\right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}}\right)^2 = 1$$

definierar en två-mantlad hyperboloid med medelpunkt P där

$$\begin{aligned}\overline{OP} &= \underline{\mathbf{f}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = \left[\underline{\mathbf{f}} = \underline{\mathbf{e}} T \right] = \underline{\mathbf{e}} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = \\ &= \underline{\mathbf{e}} \begin{pmatrix} -1/2 + 1/6 + 4/3 \\ -2/6 + 4/3 \\ 1/2 + 1/6 + 4/3 \end{pmatrix}\end{aligned}$$

Ekvationen

$$-\left(y_1 - \frac{1}{\sqrt{2}}\right)^2 - \left(y_2 - \frac{1}{\sqrt{6}}\right)^2 + 2\left(y_3 - \frac{4}{\sqrt{3}}\right)^2 = 1$$

definierar en två-mantlad hyperboloid med medelpunkt P där

$$\begin{aligned}\overline{OP} &= \underline{\mathbf{f}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = [\underline{\mathbf{f}} = \underline{\mathbf{e}} T] = \underline{\mathbf{e}} \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{6} \\ 4/\sqrt{3} \end{pmatrix} = \\ &= \underline{\mathbf{e}} \begin{pmatrix} -1/2 + 1/6 + 4/3 \\ -2/6 + 4/3 \\ 1/2 + 1/6 + 4/3 \end{pmatrix} = \underline{\mathbf{e}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\end{aligned}$$