**Exempel:** Approximera $e$ med ett rationellt tal $r$ s.a. $|e-r| \leq 10^{-2}$.

**Lösning:**

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \qquad e = e^1 = \sum_{n=0}^{\infty} \frac{1^n}{n!} = \underbrace{\sum_{n=0}^{N} \frac{1}{n!}}_{\text{Approx.}} + \underbrace{\sum_{n=N+1}^{\infty} \frac{1}{n!}}_{\text{rest.}}$$

$$\sum_{n=N+1}^{\infty} \frac{1}{n!} = \frac{1}{(N+1)!}\left(1 + \frac{1}{(N+2)} + \frac{1}{(N+2)\underbrace{(N+3)}_{>N+2}} + \dots\right) \leq$$

$$\leq \frac{1}{(N+1)!}\left(1 + \frac{1}{(N+2)} + \frac{1}{(N+2)^2} + \dots\right)$$

$$\sum_{n=N+1}^{\infty} \frac{1}{n!} = \frac{1}{(N+1)!}\left(1 + \frac{1}{(N+2)} + \frac{1}{(N+2)\underbrace{(N+3)}_{>N+2}} + \dots\right) \le$$

$$\le \frac{1}{(N+1)!}\left(1+\frac{1}{(N+2)} + \frac{1}{(N+2)^2} + \dots\right) =$$

$$= \frac{1}{(N+1)!}\sum_{n=0}^{\infty} \left(\frac{1}{N+2}\right)^n = \Big/ \sum_{n=0}^{\infty} q^n = \frac{1}{1-q} \Big/$$

$$= \frac{1}{(N+1)!}\,\frac{1}{1-\frac{1}{N+2}} = \dots = \frac{1}{(N+1)!}\,\frac{N+2}{N+1}$$

$N=4$ ger $\quad \frac{1}{(4+1)!}\,\frac{4+2}{4+1} = \frac{1}{100} \approx 10^{-2}.$

$$r = \sum_{n=1}^{4} \frac{1}{n!} = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = \underline{\underline{\frac{65}{24} = r}}.$$