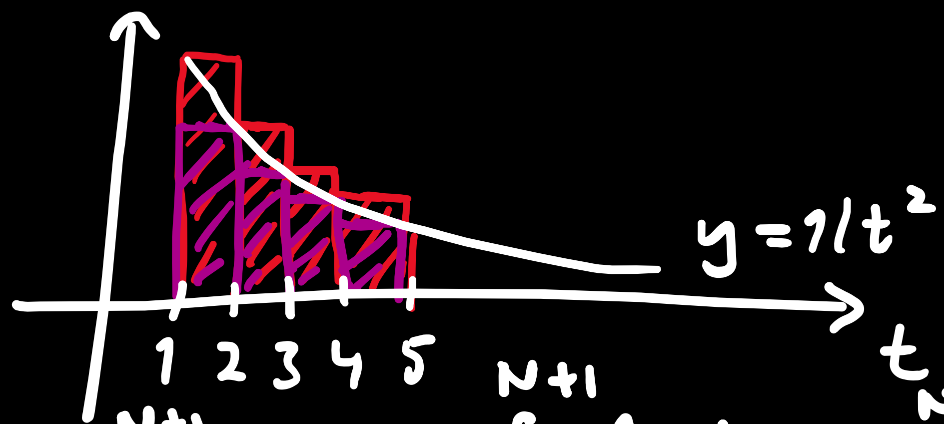
Exempel: Visa att $1 \le \sum_{k=1}^{\infty} \frac{1}{k^2} \le 2.$

Lösning:



$$\int_1^5 \frac{1}{t^2}\,dt \le \sum_{k=1}^{4} \frac{1}{k^2}$$

$$\int_1^5 \frac{1}{t^2}\,dt \ge \sum_{k=2}^{5} \frac{1}{k^2}$$

$$\sum_{k=2}^{N+1} \frac{1}{k^2} \le \int_1^{N+1} \frac{1}{t^2}\,dt \le \sum_{k=1}^{N} \frac{1}{k^2} \Rightarrow \sum_{k=2}^{\infty} \frac{1}{k^2} \le \int_1^{\infty} \frac{1}{t^2}\,dt \le \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\int_1^\infty \frac{1}{t^2}\,dt = \lim_{T\to\infty} \int_1^T \frac{1}{t^2}\,dt = \lim_{T\to\infty} \left[\frac{-1}{t}\right]_1^T = \underline{1}.$$

$$\underline{1} = \int_1^\infty \frac{1}{t^2}\,dt \le \underline{\sum_{k=1}^\infty \frac{1}{k^2}} = 1 + \sum_{k=2}^\infty \frac{1}{k^2} \,\underline{\le}\, 1 + \int_1^\infty \frac{1}{t^2}\,dt = \underline{2}.$$