Sammanfattning av formler för kroklinjiga koordinater

Gradienten ges av:

$$\nabla\Phi(u,v,w) = \hat{u}\frac{1}{h_u}\frac{\partial\Phi}{\partial u} + \hat{v}\frac{1}{h_v}\frac{\partial\Phi}{\partial v} + \hat{w}\frac{1}{h_w}\frac{\partial\Phi}{\partial w}.$$

För vektorfältet $\mathbf{A} = A_u\hat{u} + A_v\hat{v} + A_w\hat{w}$ har vi följande formler:

$$\operatorname{div}\mathbf{A} = \nabla\cdot\mathbf{A} = \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}(A_u h_v h_w) + \frac{\partial}{\partial v}(A_v h_u h_w) + \frac{\partial}{\partial w}(A_w h_u h_v)\right]$$

$$\operatorname{rot}\mathbf{A} = \nabla\times\mathbf{A} = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\hat{u} & h_v\hat{v} & h_w\hat{w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_u A_u & h_v A_v & h_w A_w \end{vmatrix}$$

För cylinderkoordinater: med $(u,v,w) = (\rho,\phi,z)$ har vi:

$$h_u = h_\rho = 1, \quad h_v = h_\phi = \rho, \quad h_w = h_z = 1.$$

$$\mathbf{r}(\rho,\phi,z) = \rho\,\hat{\rho} + z\,\hat{z}.$$

$$\hat{\rho} = \frac{1}{h_\rho}\frac{\partial\mathbf{r}}{\partial\rho} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$
$$\hat{\phi} = \frac{1}{h_\phi}\frac{\partial\mathbf{r}}{\partial\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$
$$\hat{z} = \frac{1}{h_z}\frac{\partial\mathbf{r}}{\partial z} = \hat{z}.$$

För sfäriska koordinater: med $(u,v,w) = (r,\theta,\phi)$ har vi:

$$h_u = h_r = 1, \quad h_v = h_\theta = r, \quad h_w = h_\phi = r\sin\theta.$$

$$\mathbf{r}(r,\theta,\phi) = r\,\hat{r}.$$

$$\hat{r} = \frac{1}{h_r}\frac{\partial\mathbf{r}}{\partial r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{\theta} = \frac{1}{h_\theta}\frac{\partial\mathbf{r}}{\partial\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{\phi} = \frac{1}{h_\phi}\frac{\partial\mathbf{r}}{\partial\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}.$$

Vektorformler

1. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
2. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
3. $\nabla(\alpha\Phi + \beta\Psi) = \alpha\nabla\Phi + \beta\nabla\Psi$
4. $\nabla \cdot (\alpha\mathbf{A} + \beta\mathbf{B}) = \alpha\nabla \cdot \mathbf{A} + \beta\nabla \cdot \mathbf{B}$
5. $\nabla \times (\alpha\mathbf{A} + \beta\mathbf{B}) = \alpha\nabla \times \mathbf{A} + \beta\nabla \times \mathbf{B}$
6. $\nabla(\Phi\Psi) = (\nabla\Phi)\Psi + \Phi(\nabla\Psi)$
7. $\nabla \cdot (\Phi\mathbf{A}) = (\nabla\Phi) \cdot \mathbf{A} + \Phi(\nabla \cdot \mathbf{A})$
8. $\nabla \times (\Phi\mathbf{A}) = (\nabla\Phi) \times \mathbf{A} + \Phi(\nabla \times \mathbf{A})$
9. $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$
10. $\nabla \cdot (\nabla\Phi) = \dfrac{\partial^2\Phi}{\partial x^2} + \dfrac{\partial^2\Phi}{\partial y^2} + \dfrac{\partial^2\Phi}{\partial z^2}$ i kartesiska koordinater
11. $\nabla \times (\nabla\Phi) = 0$ för alla Φ
12. $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

Dessa formler gäller för alla konstanter α, β, alla deriverbara skalärfält Φ, Ψ och alla deriverbara vektorfält $\mathbf{A}$, $\mathbf{B}$.