Hand-in Exercises 1, TATA49 Geometry with Applications

Maple or Matlab may be used in the calculations.

The exercises are customized by your birthdays as follows: p_1, p_2 is the month of your birthday, where January is 01, February 02,..., and December 12. q_1, q_2 is the date in the month, between 11 and 29, nearest to your birthday. r_1, r_2 are the two last digits in the year you were born. However, if you were born in 1990, then $r_1, r_2 = 9, 1$; if you are born in 2000 $r_1 = 9, r_2 = 4$.

Exercise 1 *Determine the affine transformation that takes $P(p_2+3, p_2+2)$ to $P'(0,0)$; $Q(q_1+p_1, q_2+1)$ to $Q'(1,1)]$, and $R(r_1, 0)$ to $R'[(0,1)$. Decompose it as a product of a similarity, a strain and a shear. Is there any fixed point or line by the transformation?*

Exercise 2 **a** *Show that a similarity transforms a circle in a circle. What is the relation between the radius of the circles?*

b *Show that a similarity transforms an ellipse, hyperbola respectively in another ellipse (hyperbola) of the same eccentricity.*

Exercise 3 *Determine the isometry(ies), if existing, that takes the line $l[r_1, r_2, p_1+p_2]$ to the line $l'[1,1,0]$, the line $m[r_1, r_2, p_1+p_2+2]$ to $m'[1, 1, \frac{2\sqrt{2}}{\sqrt{r_1^2+r_2^2}}]$ and $n[r_2, -r_1, \frac{r_2(p_1+p_2)}{r_1}]$ to $n'[1,-1,0]$.*

Exercise 4 *Consider the quadrangle with corners $P(r_1, p_1+q_2, 1), Q(r_1+4, p_1+q_2+2, 1), R(r_1+3, p_1+q_2+4, 1), Q(r_1-1, p_1+q_2+2, 1)$.*

1. *Which kind of quadrangle is it? Determine its sides and angles.*

2. *Show that there is an similarity transforming the quadrangle above to the one with corners $A(r_2, p_2+q_1, 1), B(r_2+5, p_2+q_1, 1), C(r_2+4, p_2+q_1-2, 1), D(r_2+1, p_2+q_1+2, 1)$.*

3. *Give the matrix (matrices) of this/these similarity(ies), and its ratio.*

Exercise 5 *Consider the isometry that fixes the points $P(r_2, r_1-p_1)$ and $Q(q_2, q_1+p_2)$. Describe it geometrically, completely.*

Exercise 6 *Determine all the indirect isometries that fix each of the lines $l_a[p_1+r_2, r_1-q_1, a]$, $a \in \mathbb{R}$. Describe them*

Exercise 7 *Determine three indirect isometries taking the point $P(p_1, p_2)$ to the point $P'(r_1+q_1, r_2+q_2)$. What can you see on the axis of the isometries?*

Exercise 8 *Give three different rotations that takes the point $P(p_1, p_2)$ to the point $P'(r_1+q_1, r_2+q_2)$. Determine the geometric locus form by the centres of the rotations.*

Exercise 9 **A)** *Show that any direct isometry T of the plane can be expressed in terms of complex numbers as $T(z) = az + v$, where $|a| = 1$, $v = v_1 + iv_2$, $z = x + iy$.*

B) *Give the matrix of the isometry above.*

Exercise 10 *Show that the isometry product $S_{l_3}S_{l_2}S_{l_1}$ of the reflections in lines $l_1[0,1,0], l_2[\sqrt{3},-1,\sqrt{3}], l_3[\sqrt{3},1,-\sqrt{3}]$ is a glide-reflection. Determine its axis and translation vector.*

Exercise 11 *Consider the affine transformation T with matrix*
$$\begin{pmatrix} \frac{r_2}{r_1^2+r_2^2} & \frac{r_1}{r_1^2+r_2^2} & 1-\frac{r_2}{r_1^2+r_2^2} \\ \frac{r_1}{r_1^2+r_2^2} & -\frac{r_2}{r_1^2+r_2^2} & -2\frac{r_1}{r_1^2+r_2^2} \\ 0 & 0 & 1 \end{pmatrix}.$$
Describe it geometrically.

Exercise 12 *Give conditions to the different centres and angles of two rotations such that the product of the two rotations is a translation. What is the vector of the resulting translation?*

Exercise 13 *In the figure below you see the arm of a planar robot. Give the function f that determines the configuration (position and orientation, total angle) of the hand depending on the parameters of the arm.*

Exercise 14 *With the robot arm as in the figure below, determine, if possible, the parameters for the angles at the junctions (links) the controller should get if the coordinates of the point are $P_{(H)}(\sqrt{2}/2, \sqrt{2}/2)$, relative to the local coordinates (x_H, y_H) at the hand, and $P_0(1+\sqrt{2}, 2)$, relative to the coordinates (x_0, y_0) for the controller. The total angle for the hand (the part of the arm with the hand) is π rad, and the segments all have length 1.*