Hand-in Exercises 1, TATA49 Geometry with Applications. Fall 2021

Maple or Matlab may be used in the calculations.

The exercises are customized by your birthdays as follows: p_1, p_2 is the month of your birthday, where January is 01, February 02,..., and December 12. q_1, q_2 is the date in the month, between 13 and 29, nearest to your birthday. r_1, r_2 are the two last digits in the year you were born. However, if you were born in 1990, then $r_1, r_2 = 9, 1$; if you are born in 200i $r_1 = 5 + i, r_2 = 4 + i$.

Exercise 1 *Determine the affine transformation that takes the line $l[1, 0, 1]$ to $l'[p_1, p_2, 1]$; the line $q[0, 1, 1]$ to $q'[p_1 + q_2, p_2 + p_1, 1]$, and $r[1, 1, 1]$ to $r'[r_2, r_1, 1]$. Decompose it as a product of a similarity, a strain and a shear. Is there any fixed point or line by the transformation?*

Exercise 2 **a** *Show that a similarity transforms a circle in a circle. What is the relation between the radius of the circles?*

b *Show that a similarity transforms an ellipse, hyperbola respectively in another ellipse (hyperbola) of the same eccentricity. Recall that if a hyperbola is defined by a symmetric matrix C with eigenvalues $\lambda_1 > 0$ and $\lambda_2 < 0$, then the eccentricity e is defined by $e = \sqrt{1 + \frac{\lambda_1}{|\lambda_2|}}$.*

Exercise 3 *Determine the isometry that takes the points $P(p_1, p_2)$ and $Q(q_1, q_2)$ to $P'(r_1, r_2)$ and $Q'(r_1 + \frac{p_1\sqrt{(p_1-q_1)^2+(p_2-q_2)^2}}{\sqrt{p_1^2+p_2^2}}, r_2 + \frac{p_2\sqrt{(p_1-q_1)^2+(p_2-q_2)^2}}{\sqrt{p_1^2+p_2^2}})$ respectively.*

Exercise 4 **1.** *Are the points $P(r_2, p_1), Q(r_1, q_1), R((1+q_2)r_2 - q_2 r_1, (1+q_2)p_1 - q_2 q_1)$ collinear? In that case determine the equation of the line.*

2. *Are the lines $l[r_2, r_1, 1], m[q_1, -q_2, 0], n[r_2^2 + p_2 q_1, r_2 r_1 - q_2 p_2, r_2]$ concurrent? In that case determine the coordinates of the point.*

3. *Determine the distance from the point in* **2.** *to the line in* **1.**.

Exercise 5 *Determine the images of the points and lines in Exercise 4 above under the affine transformation with matrix* $\begin{pmatrix} r_2+1 & r_2+r_1 & q_1 \\ r_1+1 & r_1+1 & q_2 \\ 0 & 0 & 1 \end{pmatrix}$.

Exercise 6 *Show that an isometry f which satisfies that $f^2 = 1_d$ is a reflection or a rotation with angle π.*

Exercise 7 *Determine all the direct isometries that fix each of the lines $l_a[a, \sqrt{1-a^2}, -ar_1 - \sqrt{1-a^2}p_2]$, $-1 \le a \le 1$. Describe them. Are the lines concurrent?*

Exercise 8 *Identify the indirect isometries taking the point $P(q_1, q_2 + q_1)$ to the point $P'(r_2 + q_1, r_1 + q_1)$. What can you see on the axis of the isometries?*

Exercise 9 *Identify the direct isometries that take the point* $P(q_1, q_2 + q_1)$ *to the point* $P'(r_2 + q_1, r_1 + q_1)$. *Determine the geometric locus form by the centres of the rotations.*

Exercise 10 **A)** *Show that any direct isometry* T *of the plane can be expressed in terms of complex numbers as* $T(z) = az + v$, *where* $|a| = 1$, $v = v_1 + iv_2$, $z = x + iy$.

B) *Give the matrix of the isometry above.*

Exercise 11 *Determine the similarity that takes the points* $Pp_1, p_2, Q(q_1, q_2)$ *to the points* $P'(r_1, r_2)$ *and* $Q'(r_1 + p_1, r_2 + p_2)$ *respectively. What is the ratio of the similarity?*

Exercise 12 *Let* f *be a glide-reflection. Show that* f^2 *is a translation. What is the vector of the translation?*

Exercise 13 *Consider the affine transformation* T *with matrix* $\begin{pmatrix} \frac{1+k}{2} + \frac{1-k}{2}a & \frac{1-k}{2}b & 0 \\ \frac{1-k}{2}b & \frac{1+k}{2} - \frac{1-k}{2}a & 0 \\ 0 & 0 & 1 \end{pmatrix}$, *where* $k = r_2$, *and* a, b *satisfy* $a^2 + b^2 = 1$. *Describe it geometrically. Is there an axis for the transformation? Which one?*

Exercise 14

1. *In the figure below you see the arm of a planar robot. Give the function* f *that determines the configuration (position and orientation, total angle) of the hand depending on the parameters of the arm. The length of the fixed rigid link is* $l_1 = 8$ *units, the length of the rigid link starting at* J_2 *is* $l_2 = 4$, *and the length of the rigid link to the hand is* $l_3 = 4$.

2. *With the robot arm as in the figure below, determine, if possible, the parameters for the angles at the junctions (links) the controller should get if the coordinates of the point are* $P_{(H)}(-4, 2\sqrt{3})$, *relative to the local coordinates* (x_H, y_H) *at the hand, and* $P_0(-\sqrt{3}, 7)$, *relative to the coordinates* (x_0, y_0) *for the controller. The total angle for the hand (the part of the arm with the hand) is* $2\pi/3$ *rad, and the segments all have length 1.*