Hand-in Exercises 1, TATA49 Geometry with Applications. Fall 2023

Maple or Matlab may/should be used in the calculations.

The exercises are customized by your birthdays as follows: p_1, p_2 is the month of your birthday, where January is 01, February 02,..., and December 12. q_1, q_2 is the date in the month, between 13 and 29, nearest to your birthday. r_1, r_2 are the two last digits in the year you were born. However, if you were born in 1990, then $r_1, r_2 = 9, 1$; if you are born in 200i $r_1 = 5 + i, r_2 = 4 + i$.

Exercise 1 *Determine the affine transformation that takes the line* $l[1, 1, 1]$ *to* $l'[p_1 + 3, p_2 + 3, 1]$*; the line* $q[2, 4, 1]$ *to* $q'[p_1 + q_1, p_2 + q_2, 1]$*, and* $r[3, -1, 1]$ *to* $r'[r_2 + p_1, r_1 + p_2, 1]$*. Decompose it as a product of an isometry, a dilation, a strain and a shear. Is there any fixed point or line by the transformation?*

Exercise 2 **a** *Determine the image of the equilateral hyperbola with equation* $x_1x_2 = 1$ *under the affine transformation with matrix* $\begin{pmatrix} r_2 & r_2 + p_1 & 2q_1 \\ r_1 + p_1 & r_1 + p_2 & 3q_2 \\ 0 & 0 & 1 \end{pmatrix}$*. Determine the eccentricities.*

b *Show that a similarity transforms an ellipse, hyperbola respectively in another ellipse (hyperbola) of the same eccentricity. Recall that if a hyperbola is defined by a symmetric matrix* C *with eigenvalues* $\lambda_1 > 0$ *and* $\lambda_2 < 0$*, then the eccentricity* e *is defined by* $e = \sqrt{1 + \frac{\lambda_1}{|\lambda_2|}}$*. The eccentricity of an ellipse is given by* $e = \sqrt{1 - \frac{\lambda_1}{|\lambda_2|}}$*, where* $0 < \lambda_1 < \lambda_2$*.*

Exercise 3 *Determine the isometry that takes the points* $P(r_2 + p_1, r_1 + p_2)$ *and* $Q(p_1 + q_1, p_2 + q_2)$ *to* $P'(r_1, r_2)$ *and* $Q'(r_1 + \frac{q_1\sqrt{(r_2-q_1)^2+(r_1-q_2)^2}}{\sqrt{q_1^2+q_2^2}}, r_2 + \frac{q_2\sqrt{(r_2-q_1)^2+(r_1-q_2)^2}}{\sqrt{q_1^2+q_2^2}})$ *respectively.* ***The*** *isometry?*

Exercise 4 **1.** *Are the points* $P(r_2 + p_1, r_1 + p_2), Q(p_1 + q_1, p_2 + q_2), R(r_1r_2 + q_1(1 - r_1) + p_1, (r_1^2 + q_2(1 - r_1) + p_2)$ *collinear? In that case determine the equation of the line.*

2. *Are the lines* $l[r_2 + q_1, r_1 + q_2, 1], m[p_1, -p_2, 0], n[p_1(r_2 + 2q_1 + 2) + 3r_2 + 3q_1, p_1(r_1+q_2)-p_2(q_1+2)+3r_1+3q_2, p_1+3]$ *concurrent? In that case determine the coordinates of the intersection point.*

3. *Determine the distance between the images by the affinity in Exercise 1 of the point in* **2.** *and the line in* **1.***.*

Exercise 5 *Determine the images of the points and lines in Exercise 4 above under the affine transformation given in Exercise 1*

Exercise 6 *Show that an isometry* f*, not the identity, which satisfies that* $f^2 = 1_d$ *is a reflection or a rotation with angle* π*.*

Exercise 7 *Determine all the direct isometries that fix each of the lines* $l_a[r_1+p_1, r_2+q_2, ar_2]$, $0 \leq a$. *Describe them. Are the lines concurrent?*

Exercise 8 *Identify the indirect isometries taking the point* $P(r_2+p_1+q_1, r_1+p_2+q_2)$ *to the point* $P'(p_1+q_2+r_2, p_2+q_1+r_1)$. *Determine the geometric locus formed by the axis of the isometries.*

Exercise 9 *Identify the direct isometries that take the point* $P(r_2+p_1+q_1, r_1+p_2+q_2)$ *to the point* $P'(p_1+q_2+r_2, p_2+q_1+r_1)$. *Determine the geometric locus formed by the centres of the rotations.*
Compare geometrically the geometric loci given in Exercises 8 and 9.

Exercise 10 **A)** *Show that any direct similarity* T *of the plane can be expressed in terms of complex numbers as* $T(z) = az + v$, *where* $|a| = r$, $v = v_1 + iv_2$, $z = x + iy$.
B) *Give the matrix of the similarity above.*

Exercise 11 *Determine the similarity that takes the points* $P(r_2+p_1+q_1, r_1+p_2+q_2), Q(r_2+p_2, r_1+q_2)$ *to the points* $P'(p_1+q_2+r_2, p_2+q_1+r_1)$ *and* $Q'(r_1+p_1, r_2+q_1)$ *respectively. What is the ratio of the similarity?*

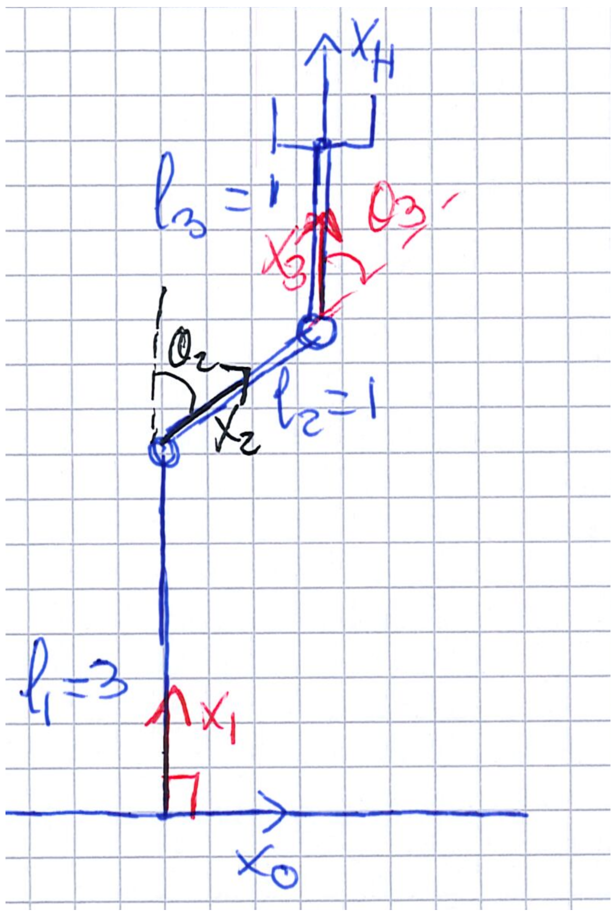
Exercise 12 *Give the fractal formed by the union images of* $\{(0,0)\}$ *under a sequence of compositions of these three similarity contractions:* f_1 *is a dilation of ratio* $\frac{2}{3}$ *followed by a rotation of angle* $\frac{2\pi}{r_2+1}$, *and a translation of vector* $\mathbf{v}_1 = (0,0)$*;* f_2 *is a dilation of ratio* $\frac{1}{2}$ *followed by a rotation of angle* $\frac{2\pi}{r_2}$, *and a translation of vector* $\mathbf{v}_1 = (1,0)$, *and* f_3 *is a dilation of ratio* $\frac{2}{3}$ *followed by a rotation of angle* $\frac{2\pi}{r_1-1}$, *and a translation of vector* $\mathbf{v}_1 = (0,1)$. *The probabilities for the appearances of* f_1, f_2, f_3 *are 0.3, 0.4 and 0.3 respectively. You may use the maple commands given in the examples of fractals.*

Exercise 13 *Consider the affine transformation* T *with matrix*
$$\begin{pmatrix} \frac{q_1}{\sqrt{q_1^2+q_2^2}} & \frac{q_2}{\sqrt{q_1^2+q_2^2}} & \frac{r_1(\sqrt{q_1^2+q_2^2}-q_1)-r_2q_2}{\sqrt{q_1^2+q_2^2}} \\ \frac{q_2}{\sqrt{q_1^2+q_2^2}} & \frac{-q_1}{\sqrt{q_1^2+q_2^2}} & \frac{r_2(\sqrt{q_1^2+q_2^2}+q_1)-r_1q_2}{\sqrt{q_1^2+q_2^2}} \\ 0 & 0 & 1 \end{pmatrix}.$$
Describe it geometrically and identify it. Is there an axis for the transformation? If existing, identify the axis

Exercise 14

1. *In the figure below you see the arm of a planar robot. Give the function* f *that determines the configuration (position and orientation, total angle) of the hand depending on the parameters of the arm. The lengths of the fixed rigid links are* $l_1 = 3, l_2 = 1, l_3 = 1$ *units.*

2. *With the robot arm as in the figure below, determine, if possible, the parameters for the angles at the junctions (links) the controller should get if the coordinates of the point are* $P_{(H)}(1/2, \sqrt{3}/2)$, *relative to the local coordinates* (x_H, y_H) *at the hand, and* $P_0(-2,3)$, *relative to the coordinates* (x_0, y_0) *for the controller, and total angle for the hand (the part of the arm with the hand) is* π *rad. Observe that it could be no solution.*



X_H
$l_3 = 1$
θ_3
X_3
θ_2
$l_2 = 1$
X_2
$l_1 = 3$
X_1
X_0